European City^2: Computational Social Science Simulation for Democracy

Topic: HORIZON-CL2-2024-DEMOCRACY-01-06 | Type of Action: HORIZON-RIA
Start: 01/01/2025 | Duration: 36 months | End: 31/12/2027

Deliverable Title	Literature Review Paper
Deliverable Number	D2.2
Work Package (WP)	WP2 Literature Review
Lead Beneficiary	11 EISM (European Institute of Science in Management)
Dissemination Level	PU
Nature	R (Report)
Due Date	31/08/2025
Submission Date	02/09/2025
Document Version	v1.0

Main Authors	Luis Razo (EISM), Michael Suleymanov (BIU), Gal Amit, (BIU), Eliahu Cohen (BIU)
Contributors	Iza Romanowska (AU), Jan Lorenz, (CUB), Michal Fabinger, (Fabinger), Rafael Hortala-Vallve (LSE), Kristoffer Laigaard Nielbo (AU)
Internal Reviewers	Nikola Grunchevski (HYB), Lily Yakova (CSD), Leonor Gil, (EISM)

Version	**Date**	**Comments**
1.0	1/09/2025	Deliverable turned in

European
Commission

Funded by the European Union (Grant Agreement No. 101178170). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the granting authority can be held responsible for them.

A Critical Review of Voting Systems: Recovering Subjective Value by Bridging Classical and Quantum Quadratic Voting

European City Squared Project - Deliverable 2.2 - Literature Review Paper

Abstract

Modern democratic theory is built upon a foundational blind spot: it treats political value as objective and evenly distributed, overlooking the subjective nature of preference intensity and civic commitment. While economic theory has long embraced subjective value, voting systems continue to flatten individual differences into the numerical equality of "one person, one vote." This paper traces the historical origins, structural consequences, and computational pathways to potentially resolve this disconnect. We recover a neglected European lineage of relational subjective value, contrasting it with the Enlightenment's shift toward deterministic egalitarianism and its well-known voting impossibility theorems. Recent innovations such as Qualitative Voting, Quadratic Voting, and Quantum Voting represent early attempts to account for preference intensity. Advancing this trajectory, we present a formal computational framework that bridges quantum aggregation dynamics and classical agent-based modeling through a scalable "wormhole" interface, producing a fairer, bidirectional framework of relational subjective value, grounded in mutual accountability and the reciprocal reinforcement of citizen commitment and institutional trust. This approach bypasses Arrow's impossibility constraints, leads to theoretically more resilient democracies, and serves as a cross-disciplinary bridge to experiment with the quantum-classical divide in computational social science. We conclude by examining Denmark's Samfundssind as a real-world illustration of civic trust's institutional scalability, anchoring the theoretical contributions of this work in practical governance contexts.

1 Introduction

Although both voting theory and economic value theory are concerned with how individual valuations are expressed within collective decision-making, their intellectual trajectories have evolved in striking isolation. Economic theory rigorously established the subjective theory of value, recognizing value as inherently relational, context-dependent, and grounded in individual perception and consent (Grice-Hutchinson, 1952). Yet, modern democratic systems, deeply entrenched in the principle of "one person, one vote," implicitly assume an objective and evenly shared collective good—a flattening of political value that stands in stark contrast to economic theory's embrace of subjective pluralism. Cultural traditions like the Danish concept of Samfundssind (literal: 'societal-mindedness') – which emphasize civic responsibility and trust – demonstrate how societies can informally compensate for this design flaw, fostering collective outcomes where individual commitment plays a vital role (Svendsen and Svendsen, 2016; The Guardian, 2024). This divide between political voice and subjective value—what we identify as a structural blind spot in democratic theory—is not merely an oversight but a deeper failure to account for the fact that individuals bring unequal preferences, commitments, and stakes to public decisions. We argue that these differences must be formally represented if democratic systems are to withstand the growing number of threats to democracy (European Commission, 2023; Freedom House, 2023).

At its core, this paper examines a historical divergence: Should collective decisions emerge solely from the aggregation of individual self-interest, as Adam Smith's "Invisible Hand" and his departure from European tradition suggests (Hirschman, 1977; Posner and Weyl, 2018)? Or should they incorporate mechanisms that weigh judgment, contribution, and civic-mindedness, as envisioned by Aristotle (Aristotle, 1996), Ramon Llull (Colomer, 2012; Hägele and Pukelsheim, 2001; McLean and Urken, 1995), and the School of Salamanca (Grice-Hutchinson, 1952; Vitoria, 1991; Tierney, 1997)? Rather than settle this question philosophically, we approach it computationally: by reviewing voting systems that explicitly embed subjective valuations, enabling societies to decide how to balance self-interest, civic input, and collective wisdom through transparent, value-sensitive algorithms.

1.1 Context and Motivation

The ongoing crises of democratic legitimacy—rising polarization, declining trust, voter manipulation, and resurgent authoritarianism (Freedom House, 2023; Ullah, 2025; Markowski and Kotnarowski, 2025)—stem not only from poor implementation but from a design flaw: the assumption of equal voice in the face of unequal preference intensity and civic engagement. Reconnecting democratic design to a subjective theory of value offers a possible way forward, one that aligns procedural equality with the lived realities of democratic participation and justice.

1.2 Scope and Interdisciplinary Approach

This paper provides a critical review that draws from political theory, social choice, economics, neuroscience, quantum mechanics, and computational social science. We assess the limitations of classical voting frameworks, recover overlooked intellectual traditions, and explore how expressive mechanisms—such as Qualitative Voting, Quadratic Voting, and Quantum Voting—attempt to incorporate subjective valuation into aggregation design. The review proceeds as follows: Section 2 outlines foundational concepts and normative objectives of electoral systems. Sections 3 and 4 examine the operational limits of classical systems. Sections 5 and 6 assess expressive and quantum innovations. Section 7 reconnects historical frameworks to modern computational design. Sections 8 through 10 synthesize emergent models like Quantum Quadratic Voting, to be validated through agent-based simulations and pilot testing in Denmark.

1.3 Empirical Grounding and Forward Outlook

While traditions like Samfundssind enable egalitarian systems to function in high-trust societies, such norms are not present universally. Nations differ in their level of so-called “civic culture” or “civic duty” and these cultural traits are persistent across generations. Denmark’s success with one-person-one-vote may stem less from the rule itself and more from the civic trust that sustains it. This view is supported by findings from the OECD’s Trustlab experiment, which shows that trust in government is shaped by institutional integrity, responsiveness, and service quality (Murtin et al., 2018; Edelman, 2024; Mozumder, 2022). These dynamics point toward a deeper insight: legitimacy may depend not only on how individuals evaluate institutions, but also on how systems account for varying levels of individual civic contribution—suggesting a more reciprocal and value-sensitive model of democratic trust. Yet even in historically stable Nordic democracies, high-trust norms are not immune to erosion, as recent democratic backsliding and polarization in Sweden and Finland suggest (Tørnqvist, 2023). This risk underscores why the challenge is fundamentally computational and requires formal methods for capturing and weighting such contributions within collective decisions.

In the sections that follow, we show how quantum-enhanced quadratic voting provides one computational pathway for doing so. The idea of weighting civic commitment—measurable contributions such as voluntary giving, public service, or other pro-social acts—as part of collective decision-making is absent from the history of democratic thought. From Plato to Rousseau to Rawls, theories of representation have debated fairness and equality but have not considered civic investment itself as a basis for political influence, defaulting instead to egalitarian or majoritarian axioms. Even advanced voting systems, from Condorcet’s pairwise rankings to modern Quadratic Voting, stop short of linking decision-making weight to tangible contributions to the common good. This review examines whether this omission is optimal from both normative and institutional perspectives.

2 Foundations of Electoral Systems

Electoral systems are institutional mechanisms designed to efficiently convert individual preferences into collective decisions that are perceived as legitimate and fair. To evaluate and design such systems effectively, it is essential to understand several foundational concepts, including preference aggregation, legitimacy, strategic behavior, and preference intensity—each of which shapes the quality and resilience of democratic outcomes.

Preference Aggregation: At the heart of electoral systems lies the challenge of preference aggregation: the process by which individual preferences are combined to arrive at a collective decision (List, 2013). Whether in the context of political elections, referenda, or committee choices, preference aggregation seeks to translate a diverse array of individual opinions, priorities, and stakes into a single, actionable outcome. The design of aggregation mechanisms profoundly influences not only who wins, but how fairly and accurately the outcome reflects the underlying distribution of societal preferences.

Legitimacy: Procedural and Substantive: The concept of legitimacy is central to evaluating electoral systems. Procedural legitimacy refers to the perceived fairness of the rules and processes by which decisions are made—often associated with principles like equal voice, transparency, and due process (Tyler, 2006). Substantive legitimacy, by contrast, concerns the extent to which collective decisions are perceived as just, representative, or aligned with the public good, regardless of the procedural path taken to reach them (Peter, 2023; Scharpf, 1999). An electoral system may be procedurally legitimate but substantively deficient if it produces outcomes that systematically misrepresent or disregard significant portions of societal preferences or stakes.

Strategic Behavior and Manipulation: A well-known challenge in voting system design is the prevalence of strategic behavior, where voters may have incentives to cast insincere ballots to influence the outcome more effectively (Taylor, 2005). The Gibbard–Satterthwaite theorem formalizes the inevitability of strategic manipulation in any deterministic voting system with three or more alternatives that seeks to aggregate preferences in a non-dictatorial way (Gibbard, 1973; Satterthwaite, 1975). This introduces persistent tensions between fairness, expressiveness, and strategic resistance in the design of voting mechanisms.

Preference Intensity: A Neglected Dimension: Most traditional voting systems, including plurality and majoritarian rules, implicitly treat all preferences as equal in weight, focusing solely on ordinal rankings or binary choices. However, individuals may care about outcomes with differing levels of intensity. Preference intensity refers to the degree of importance or urgency an individual assigns to a particular outcome. While mechanisms like Quadratic Voting explicitly incorporate this dimension by allowing voters to allocate votes in proportion to their intensity of preference, and certain systems—such as cumulative voting in some open-list proportional elections (e.g., Switzerland, several German Länder) or Majority Judgment (Balinski and Laraki, 2007)—allow voters to express strength of support, these remain exceptions. The majority of electoral systems and theoretical models still neglect this critical aspect. Ignoring preference intensity risks flattening the informational richness of collective preferences, leading to outcomes that may be procedurally fair yet substantively misaligned with societal welfare.

2.1 Historical Developments and Theoretical Results

The challenge of aggregating individual preferences into a collective decision has been a subject of intellectual inquiry for centuries. One of the earliest known attempts to address this problem was made by the 13th-century philosopher Ramon Llull, who proposed an algorithmic approach to decision-making based on exhaustive pairwise comparisons (Colomer, 2012). Llull's method, designed to ensure fairness in ecclesiastical elections, anticipated key ideas later formalized in the Condorcet method and is often regarded as a precursor to modern computational social choice. Llull's work reflects an early recognition that legitimacy in collective choice arises not from arbitrary rule, but from structured, transparent procedures that systematically compare and relate the preferences of voters.

Building on these early insights, the 18th-century philosopher and mathematician Marie Jean Antoine Nicolas de Caritat, Marquis de Condorcet, identified a fundamental paradox in majority rule systems. Known as Condorcet's paradox, it shows that collective preferences can form a cycle—where A is preferred to B, B is preferred to C, and C is preferred to A—even when each individual voter's preferences are transitive (Condorcet, 2014; Gehrlein, 2006). This finding exposed the potential incoherence of majority preference aggregation and challenged the assumption that simple majority rule necessarily produces a stable, consistent collective will.

Around the same period, Jean-Charles de Borda proposed one approach to mitigate preference cycles: the Borda count, a ranking system where voters assign ordinal rankings to all candidates, with points allocated based on position. While the Borda count reduces the likelihood of cyclic outcomes by incorporating broader preference information, it introduces its own set of vulnerabilities, including susceptibility to strategic ranking and violations of majority preference, which can undermine perceived legitimacy.

The limitations of these early systems were formalized in the 20th century by Kenneth Arrow, whose Impossibility Theorem rigorously demonstrated that no voting system can simultaneously satisfy a set of seemingly reasonable criteria—Unrestricted Domain, Non-Dictatorship, Pareto Efficiency, and Independence of Irrelevant Alternatives (IIA)—when aggregating three or more alternatives (Arrow, 1951). Arrow's result revealed a fundamental trade-off in collective decision-making, forcing system designers to prioritize among competing normative objectives. Further complicating the landscape, the Gibbard (Gibbard, 1973)–Satterthwaite (Satterthwaite, 1975) theorem extended this impossibility result to strategic behavior, proving that any deterministic voting system that selects a single winner from three or more alternatives is inherently susceptible to strategic manipulation, unless it is dictatorial or imposes severe restrictions on preference profiles. This theorem underscores the inherent tension between expressiveness and strategic resistance, highlighting the difficulty of designing systems that are both fair and robust against insincere voting.

Collectively, these foundational results have shaped the theoretical landscape of social choice, clarifying the constraints and trade-offs that must be navigated in the design of electoral systems. Condorcet (Gehrlein, 2006), Borda (Borda, 1781; Saari, 1995), Arrow (Arrow, 1951), and Gibbard (Gibbard, 1973)–Satterthwaite (Satterthwaite, 1975) each showed, in different ways, that no voting system can satisfy all desirable properties at once and that every mechanism must balance tensions between coherence, fairness, and resistance to strategic manipulation. Moreover, a voting system's effectiveness can vary with the prevailing state of polarization, consensuality, or plurality of preferences. As these dynamics shift, the fit between societal preferences and the system's aggregation rules may improve or deteriorate. It is worth noting that while such paradoxes remain foundational to social choice theory, empirical studies across a variety of electoral settings suggest they occur relatively infrequently in practice (Gehrlein, 2006; Tideman and Plassmann, 2011), highlighting what we believe to be a persistent tension between theory and practice in the field.

The tension emerges from the fact that the dominant criteria for evaluating voting systems—such as majority consistency, independence of irrelevant alternatives, and strategy-proofness—are concerned primarily with the formal fairness of the aggregation procedure, focusing on the logical consistency, impartiality, and resistance to manipulation in the decision-making process. This procedural orientation largely overlooks questions of substantive legitimacy—whether the outcomes themselves are just, representative, resilient, and aligned with the collective good—and neglects relational and value-sensitive dimensions such as preference intensity (the degree of importance or urgency an individual assigns to a particular outcome), civic contribution (measurable acts that support the common good, such as voluntary giving, public service, or other prosocial behaviors), and reciprocal valuation (the process by which such contributions are recognized and weighted by others in determining an individual's influence in collective decisions). Addressing this gap is central to the framework developed in the following section, which outlines the core objectives that should guide the design and evaluation of electoral systems.

2.2 Objectives of Electoral Systems

While the formal results of social choice theory reveal structural limitations in preference aggregation, electoral systems are ultimately judged by their ability to produce outcomes that satisfy broader normative goals. These goals are often context-dependent, but certain core objectives consistently arise in both theoretical and practical assessments of democratic decision-making. These include fairness (the consistent and transparent application of agreed-upon rules for translating individual preferences into collective outcomes) (Beetham, 1991; Lijphart, 2012), representation (the degree to which elected bodies reflect the political preferences, demographic composition, and diversity of interests present in the electorate) (Lijphart, 2012; Norris, 2014), resilience (the capacity to maintain legitimacy and performance under stress) (Lijphart, 2012; Norris, 2014), accountability (the obligation of elected officials and institutions to answer for their actions and decisions, and the capacity of voters to reward or sanction them accordingly) (Beetham, 1991; Lijphart, 2012), comprehensibility (processes are understandable and trusted by voters) (Reynolds et al., 2005), and consent (the acceptance of outcomes as legitimate by those bound by them) (Beetham, 1991; Lijphart, 2012). Together, these objectives provide a framework for evaluating the legitimacy and performance of voting systems.

Hélène Landemore's *Democratic Reason* argues that inclusive decision-making outperforms restricted models because cognitive diversity offsets individual ignorance and bias, making equal voice both fair and epistemically advantageous (Landemore, 2013). EC2 accepts the value of diversity but questions whether it alone ensures high-quality outcomes, noting that participants may differ in engagement, information, and commitment to the common good. Research shows diversity alone can still yield poor outcomes when engagement or incentives are low (Hong and Page, 2004; Lorenz et al., 2011a). EC2's reciprocal valuation principle complements diversity by weighting influence with demonstrated civic contribution. More broadly, EC2 is implemented as an agent-based modeling environment able to test any aggregation algorithm—including equal-weight diversity models, reciprocal valuation, or hybrids—under varying conditions of diversity, engagement, and strategic behavior.

Despite their importance, these objectives are rarely addressed in full by traditional electoral systems, which often prioritize formal equality at the expense of preference intensity and civic commitment. In the following section, we examine how this narrow focus on one-way aggregation limits the legitimacy and performance of collective decisions, and we introduce a conceptual framework for incorporating two-way subjective valuation into the design of democratic systems.

2.3 The Incomplete Aggregation Problem

Despite the normative goals outlined above, many traditional electoral systems focus almost exclusively on aggregating individual preferences toward collective outcomes, typically via mechanisms that ensure formal equality of input—such as one-person-one-vote. While such systems may fulfill procedural fairness, they often fail to reflect preference intensity, civic engagement, or contribution to the common good, resulting in outcomes that may be misaligned with societal welfare or perceived legitimacy. Notably, some institutional arrangements have moved away from strict equality of input to balance competing principles. For example, the European Council's qualified majority voting system weights votes by member-state population while also requiring approval from a minimum number of countries, thereby combining proportionality with state equality (Council of the European Union, 2024). This small but non-trivial departure from one-person-one-vote demonstrates that weighted influence, when grounded in transparent and agreed-upon criteria, can be compatible with democratic legitimacy.

A mathematically richer precedent comes from the Penrose square root law (Penrose, 1946), originally proposed for the United Nations General Assembly and later advanced by Poland in EU reform debates. Penrose argued that in larger countries, an individual voter's probability of changing a national outcome is smaller, and that this probability scales with the square root of the population. Therefore, to preserve equal individual influence at the supranational level, a country's voting weight should be proportional to the square root of its population. This results in a concave weighting rule—doubling voting weight requires four times the population—a logic directly analogous to QV. While not adopted in the EU, the Penrose model provides a clear, mathematically grounded example of how calibrated weighting can reconcile individual equality with aggregate proportionality.

This limitation reflects what we term the one-way subjective valuation problem. In current models, value flows in only one direction: individuals express their subjective preferences over collective outcomes, and these preferences are aggregated. What remains absent is a formal mechanism for society to value individuals in return—specifically, to account for how much each person has invested in, or depends upon, the common good. This reciprocal valuation dynamic, present in many cultural and historical forms of governance, is missing from contemporary voting systems. For example, governance traditions such as the African philosophy of ubuntu ("I am because we are") (Metz, 2011), Indonesia's gotong royong ("mutual cooperation") (Bowen, 1986), and the Andean ideal of buen vivir ("good living" through communal harmony) (Gudynas, 2011) embed legitimacy in reciprocal responsibility and shared well-being, linking individual standing in the community to demonstrated contributions to the common good.

This review identifies this gap not as a normative oversight, but as an informational limitation in aggregation design. Just as markets aggregate subjective valuations of goods to determine prices, voting systems aggregate subjective preferences to determine outcomes. However, a complete aggregation framework would also account for how much individuals contribute to the common good or are affected by collective outcomes, allowing reciprocal subjective value to inform decision-making.

Our approach conceptually frames this two-way valuation as an extension of traditional voting logic. It is value-agnostic by design, meaning the core algorithm can be adapted to any normative framework and, in our example, is capable of incorporating context-dependent civic inputs such as altruism, tax contributions, or public service. The aim is not to prescribe who should have more influence, but to give democratic systems the capacity to represent these civic dimensions transparently, should a society choose to embed them.

This conceptual structure draws on diverse traditions that view legitimacy as arising from mutual recognition, consent, and civic participation rather than mere formal equality, from the School of Salamanca's relational theory of value to governance philosophies such as ubuntu, gotong royong, and buen vivir. By operationalizing this reciprocal valuation dynamic in a computationally tractable and culturally adaptable way, we seek to provide a conceptual roadmap for future democratic design—one that expands the aggregation logic to include both individual preferences and societal recognition of civic contribution.

3 Classical Voting in Practice: Mechanisms and Operational Limitations

Classical voting systems, whether majoritarian, proportional, or deliberative, form the foundation of most democratic processes worldwide (Farrell, 2011; Reynolds et al., 2005; Fishkin, 2009). Despite their varied approaches to aggregating individual preferences into collective outcomes, they share structural weaknesses that undermine legitimacy. These weaknesses limit procedural fairness (equitable participation and transparent rules) (Dahl, 1989), substantive representativeness (alignment with diverse public preferences) (Lijphart, 2012), and electoral integrity (trust in the democratic process) (Norris, 2014). This section explores these limitations and their impact on democratic outcomes.

3.1 Shared Limitations Across Classical Systems

Building on the structural weaknesses of classical voting systems, several shared limitations undermine their legitimacy and performance. These have been widely recognized in comparative analyses of electoral design (Farrell, 2011; Norris, 2014), which highlight issues such as:

Flattening of Preference Intensity: None of these systems formally captures how deeply individuals care about particular outcomes, leading to potential mismatches between procedural fairness and substantive societal welfare (Posner and Weyl, 2018).

Vulnerability to Strategic Distortion: Whether through tactical voting, list manipulation, or gaming of deliberative structures, these systems remain susceptible to strategic behavior that can undermine fairness (Gibbard, 1973; Satterthwaite, 1975; Schumpeter, 1942).

Disconnection from Civic Contribution: Classical systems treat all votes as equal expressions of preference, irrespective of the individual's engagement, contribution, or demonstrated stake in collective goods (Putnam, 2000) (also, see Section 11 for emergent altruism).

Over-Reliance on Procedural Equality: By equating fairness with equal vote counts, these systems neglect richer informational structures—such as preference intensity and civic commitment—that are crucial for achieving outcomes perceived as legitimate and just (Posner and Weyl, 2018; Sen, 1999) (also, see Section 7 for historical context on relational valuation).

These limitations highlight the need for voting mechanisms that incorporate subjective valuation, such as preference intensity and civic contribution, into aggregation processes. The following subsections examine majoritarian, proportional, and deliberative systems, illustrating how each exhibits these limitations in practice.

3.2 Majoritarian Systems

Majoritarian systems, most notably First-Past-The-Post (FPTP), remain the default electoral mechanism in many democracies (Reynolds et al., 2005). FPTP awards victory to the candidate or option with the most votes, regardless of whether they achieve an absolute majority (Farrell, 2011). While procedurally simple (Reynolds et al., 2005), FPTP is prone to vote splitting, where similar candidates dilute each other's support (Farrell, 2011), and minority rule, where a candidate can win with far less than majority support (Reynolds et al., 2005). These dynamics encourage strategic voting, where individuals may vote insincerely to prevent a less desired outcome (Farrell, 2011).

Efforts to mitigate these flaws have produced mechanisms like Runoff Voting and Instant Runoff Voting (IRV), which require candidates to secure a majority either through sequential rounds or through ranked-choice ballots (Farrell, 2011; Reynolds et al., 2005). While these systems address some of FPTP's pathologies, they remain limited in their ability to capture preference intensity. A voter's second-choice ranking under IRV carries the same weight regardless of how marginal or fervent that preference may be (Tideman, 1995; Brams and Fishburn, 2002). Moreover, all these majoritarian systems rely on equal voice per voter, without accounting for differences in stake, contribution, or intensity of concern (Mill, 1998; Lijphart, 2012).

3.3 Proportional Representation

Proportional Representation (PR) systems aim to ensure that legislative seats are allocated in proportion to the share of votes each party or candidate receives (Farrell, 2011; Lijphart, 2012). Methods like List PR and the D'Hondt method are widely used in parliamentary systems to foster minority inclusion and reduce the distortions of majoritarianism (Lijphart, 2012). Single Transferable Vote (STV) adds a preferential ranking component, allowing votes to be redistributed to remaining candidates until all seats are filled (Farrell, 2011). While PR systems achieve greater numerical representation, they still fail to account for preference intensity—treating all votes as equally expressive regardless of how much a voter cares about an outcome (Lijphart, 2012). Moreover, strategic behaviors persist, such as candidate list manipulation and party fragmentation to exploit thresholds (Farrell, 2011). Proportional systems address some representation issues but remain limited in their ability to reflect the depth of individual valuation or civic contribution to collective goods (Mill, 1998).

3.4 Deliberative Mechanisms

In response to the limitations of aggregative voting methods, deliberative democratic mechanisms—such as citizen assemblies, mini-publics, and sortition-based panels—have gained traction (Fishkin, 2009). These systems prioritize depth of discussion and collective reasoning over direct preference aggregation, aiming to produce more reflective and considered outcomes (Dryzek, 2000a; Fishkin, 2009). By assembling a representative sample of citizens for structured deliberation, these approaches can address some of the weaknesses of mass voting, such as ignorance, polarization, and strategic distortion (Dryzek, 2000b).

Such mechanisms are not new: historical precedents include Iceland's Althing, one of the oldest parliamentary assemblies in the world (Byock, 2001), and traditional governance practices in many African and other indigenous communities, where decision-making often centered on consensus-building and extended dialogue among community members (Metz, 2011; Bowen, 1986; Gudynas, 2011). However, deliberative mechanisms face scalability challenges and often lack formalized processes for aggregating the preferences or judgments of broader populations (Fishkin, 2009). Their outcomes are typically advisory or limited in scope, failing to address the aggregation of preference intensity or reciprocal civic valuation in systematic ways (Parkinson, 2006).

Moreover, as Lorenz et al. demonstrate, even mild social influence during group deliberation can erode the informational diversity upon which collective accuracy depends (Lorenz et al., 2011b). Their experimental findings reveal that deliberation frequently induces a social influence effect (narrowing opinion diversity), a range reduction effect (compressing the spectrum of expressed views), and a confidence effect (inflating individual certainty without improving group accuracy). These distortions illustrate how aggregation mechanisms lacking relational safeguards can inadvertently amplify consensus errors, a risk that compounds with system scale and participant interdependence. This underscores the need for aggregation frameworks that balance expressiveness, strategic resistance, and diversity preservation.

4 Classical and Modern Electoral System Challenges

Efforts to address the shortcomings of traditional voting systems have produced a range of innovations, spanning improved aggregation methods to participatory and deliberative experiments. This section surveys notable electoral mechanisms across four categories: single-winner systems, multi-winner proportional methods, cardinal voting approaches, and alternative democratic innovations. While each aims to enhance fairness, representation, or deliberative depth, none fully resolves the challenges of expressing preference intensity or incorporating reciprocal civic valuation.

Single-Winner Systems: As discussed in Section 3, First-Past-The-Post (FPTP) remains dominant despite distortions like vote-splitting and minority rule. Instant Runoff Voting (IRV) introduces ranked preferences to mitigate these issues but fails to differentiate between marginal and passionate secondary preferences. Condorcet methods, which elect the candidate who would win all pairwise contests, trace their roots to Ramon Llull's 13th-century algorithmic comparisons (Colomer, 2012). However, they struggle with cyclic preferences and may fail to yield a clear winner, improving procedural fairness while remaining blind to preference intensity.

Multi-Winner and Proportional Systems: Building on Section 3's overview, systems like D'Hondt, Single Transferable Vote (STV), and List PR achieve better numerical representation of minorities and diversity through proportional seat allocation. Yet, they presume equal preference weights, remain vulnerable to strategic manipulations such as party list gaming, and overlook preference intensity or differential civic contribution—potentially enabling a structural free rider problem where minimal investors in the common good wield equal influence.

Cardinal and Non-Traditional Methods: Cardinal methods shift toward expressing degrees of approval rather than ordinal rankings. Approval Voting (Brams and Fishburn, 2007) allows endorsement of multiple candidates, while Range Voting (Smith, 2000) and STAR Voting (Equal Vote Coalition, 2019) use scoring for nuanced evaluations. These capture richer preference data but often lack safeguards against strategic exaggeration and do not link influence to civic metrics, offering partial expressivity without balancing intensity and fairness. Yet, beyond formal voting mechanisms, a crucial but often overlooked dimension in preference aggregation mechanisms is the role of bounded confidence in shaping collective outcomes. Lorenz's seminal survey on Continuous Opinion Dynamics under Bounded Confidence synthesizes models by Krause-Hegselmann and Deffuant et al., demonstrating how agents iteratively adjust their continuous opinions only when interacting with peers within a certain confidence threshold (Lorenz, 2007). These dynamics give rise to emergent phenomena such as opinion clustering, polarization, and consensus breakdown, which are structurally analogous to preference distortions observed in classical voting systems. Unlike discrete models that force binary or ordinal choices, bounded confidence frameworks preserve the continuity and granularity of individual valuations,

System Type	Fairness	Representation	Resilience	Accountability	Clarity	Consent	Captures Intensity	Includes Civic Contribution	Strategic Resistance	Notes / Limitations
Majoritarian (FPTP, IRV)	Procedurally fair in rules; outcomes may misrepresent majority	Weak minority representation	Vulnerable to shocks, polarization	Clear individual accountability	High	Moderate–High	×	×	Low–Moderate; tactical voting common	Vote splitting, minority rule, ignores intensity
Proportional Representation (List PR, STV)	Procedural fairness + better minority inclusion	High proportionality	Moderately resilient; coalition fragility	Party accountability more than individual	Moderate	Moderate–High	×	×	Moderate; list manipulation possible	Equal weighting ignores intensity; possible free rider problem
Cardinal (Approval, Range, STAR)	Procedural fairness; richer expression	Moderate–High	Context dependent	Varies	Moderate	Moderate	Partial (via scores)	×	Low–Moderate; score exaggeration possible	No civic link; vulnerable to exaggeration
Deliberative (Mini-publics, assemblies)	Fair in small-group context	Good deliberative representation; low scalability	Fragile at scale; consensus can erode diversity	High within group	Low–Moderate for public	High within group	×	×	High within group; low system-level impact	Advisory role; intensity/contribution not formalized
Qualitative Voting (QualV)	Procedural fairness; allocative expressivity	Moderate	Dependent on budget rules	Varies	Moderate	Moderate	✓ (token allocation)	×	Moderate; some strategic token allocation possible	Conceptual precursor to QV; no civic link
Quadratic Voting (QV)	Fairer alignment of influence with intensity	Moderate–High	Higher resilience to tyranny of indifferent majority	Varies; depends on context	Moderate	Moderate	✓ (quadratic cost)	×	Moderate–High; cost discourages exaggeration	Equity concerns if credits tied to money; no civic link
Quantum Voting (QVq)	Conceptual fairness via quantum relational dynamics	N/A (theoretical)	N/A	N/A	Moderate (depends on explanation)	N/A	✓ (in principle)	✓ (in principle)	N/A	Conceptual laboratory for developing quantum aggregation mechanisms; includes QQV as a specific formulation
Quantum Quadratic Voting (QQV)	Relational fairness via dynamic coherence	High	Contextually adaptive; resilient to Arrow-type constraints	Strong if transparent	Moderate	High if understood	✓ (amplitude encoding)	✓ (reciprocal civic valuation)	High; entanglement diffuses strategic incentives	Complexity in calibration and metric design; currently implemented classically in Gravitas
Gravitas (EC2 implementation of QQV)	Same fairness logic as QQV; tailored via local calibration	High	Scalable resilience via reciprocal valuation	Strong if civic metrics trusted	Moderate–High	High if participatory calibration	✓ (preference × contribution weighting)	✓ (formalized civic link)	High; ABM-tested against manipulation	Classical agent-based testbed for QQV; adaptable across cultures and institutions

Figure 1: Comparative Performance of Classical, Modern, and Novel electoral systems.

highlighting how minor variations in confidence bounds can bifurcate collective opinion landscapes. Integrating these insights into voting system design underscores the need for aggregation mechanisms that not only capture preference intensity but also account for dynamic interaction thresholds—factors that classical and many modern voting methods neglect. Lorenz's bounded confidence dynamics thus provide a critical theoretical backdrop for understanding the limitations of existing aggregation systems and motivate the exploration of expressive, context-sensitive mechanisms.

A key advance here is Qualitative Voting (QualV), proposed by Rafael Hortala-Vallve (Hortala-Vallve, 2012). QualV allocates a fixed vote budget (e.g., 10 tokens) for voters to distribute across options, enabling intensity expression by prioritizing deeply valued choices. This introduces allocative expressivity, where voters actively allocate influence based on preference strength rather than default equality—serving as a conceptual precursor to more formalized intensity mechanisms like Quadratic Voting.

Alternative Innovations: As noted in Section 3, deliberative approaches like mini-publics (Smith, 2009) and participatory budgeting foster engagement through discussion and allocation. These enhance local legitimacy but are typically advisory, scale-limited, and not designed for broad preference aggregation that encodes intensity or contribution.

Despite their advancements, classical and modern electoral systems—whether majoritarian, proportional, or cardinal—largely treat preferences as equally weighted data points, flattening individual differences in care, stake, and contribution. This limitation underscores the need for mechanisms that capture preference intensity in a fair, strategic-resistant, and computationally tractable way. In the next section, we explore Quadratic Voting (QV), an innovative approach that formalizes preference intensity through an economically principled cost mechanism, evaluating its theoretical foundations, empirical evidence, and practical implications for value-sensitive democratic design.

The analysis above highlights how each electoral system, from classical to novel designs, performs against established normative criteria while contending with persistent structural limitations. Table 1 summarizes these comparisons, integrating the evaluation criteria from Section 2 and the limitations identified in Section 3, and situating the novel mechanisms developed in this paper alongside existing approaches.

5 Preference Intensity and Quadratic Voting: Theory and Practice

Building on the expressive potential of cardinal methods like QualV, Quadratic Voting (QV) represents a more formalized approach to incorporating preference intensity into democratic decision-making. Proposed by economist Glen Weyl, QV allows voters to allocate either money or "voice credits" to express not just direction but the strength of their preferences, with the cost of votes increasing quadratically to balance intensity signaling with fairness (Weyl, 2012; Posner and Weyl, 2015). This section examines QV's theoretical foundations, empirical evidence from lab and field studies, ethical and strategic considerations, and practical implementation challenges, highlighting its pivotal role in recovering subjective value in aggregation processes.

5.1 Theoretical Foundations: Voting as a Way to Express Intensity

At its core, QV addresses the "flattening" problem in traditional voting by enabling voters to express preference intensity through a market-like mechanism. Under QV, each voter receives a fixed budget of virtual credits (e.g., 100 tokens) to purchase" votes on issues or candidates. The cost of casting v votes on a single option is v^2 credits, meaning the marginal cost rises with each additional vote. This quadratic pricing ensures that voters allocate credits proportionally to their subjective valuation, approximating marginal utility and leading to outcomes that theoretically maximize social welfare function (SWF) under certain assumptions. For a formal derivation of this quadratic cost–utility relationship, see Annex 1, which details the mathematical structure linking vote costs to welfare maximization.

Theoretically, QV draws from economic principles of costly signaling and Vickrey-Clarke-Groves (VCG) mechanisms, where the quadratic cost discourages exaggeration while allowing passionate minorities to influence outcomes without dominating them (Lalley and Weyl, 2018). In large electorates, QV converges to efficiency in equilibrium, outperforming one-person-one-vote systems by incorporating richer information about preference strength (Lalley and Weyl, 2018). Unlike ordinal rankings or simple approval, QV treats voting as a continuous expression of utility, aligning political aggregation with subjective value theory. However, this assumes symmetric information and rational behavior, limitations empirical studies have tested with mixed results; while some studies have found high voter engagement (Public Choice, 2017), other research suggests that the "hyper-rational" equilibrium required for QV is complex and that real-world voter behavior often deviates from theoretical predictions (Kaplow and Kominers, 2017).

The attraction-repulsion dynamics central to bounded confidence models provide a valuable lens for interpreting these empirical deviations. In EC2 researcher Jan Lorenz's framework, agents adjust their opinions by averaging with peers within a bounded confidence interval, resulting in emergent phenomena such as clustering, polarization, and fragmented consensus (Lorenz, 2007). This iterative process mirrors the expressive-intensity balancing that QV seeks to formalize through quadratic costs. By treating influence as a function of proximity in belief space, bounded confidence models reveal how relational thresholds and localized interactions—not hyper-rational optimization—can govern aggregation dynamics. Recognizing these mechanisms underscores the importance of designing voting systems like QV that can encode nuanced influence structures, even when voters do not behave as fully rational utility maximizers.

5.2 Empirical Review: Field Experiments, Lab Findings

Empirical evidence for QV spans laboratory experiments and real-world field trials, demonstrating its potential to enhance welfare and representation while revealing practical trade-offs. In laboratory settings, controlled experiments have shown QV's superiority in approximating efficient outcomes. For instance, a 2019 study comparing QV to storable votes and majority rule on simulated proposals found that QV increased minority victories and average welfare, though it sometimes amplified utility inequality among voters (Casella and Sanchez, 2019). The same study, using California ballot initiatives as stimuli, confirmed QV's ability to produce more welfare-optimal results than plurality voting, with bootstrapped data indicating robustness across voter samples. Recent extensions, such as a 2021 online experiment testing QV against direct democracy, reported higher participant satisfaction and reduced polarization in decision-making (Cheng et al., 2021).

Field experiments provide real-world validation. A landmark trial occurred in 2019 when the Colorado Democratic House caucus used a modified QV system with 100 virtual tokens to prioritize 30 legislative bills (Rogers, 2019). The process allocated funding more consensually, elevating minority-supported issues like mental health reforms while demoting less intensely valued ones. In decentralized governance, a 2023 study on blockchain platforms like Gitcoin showed QV (via quadratic funding) effectively balanced power, reducing plutocratic tendencies in proposal voting (Gitcoin, 2023). A 2025 case study further evaluated QV in blockchain ecosystems, finding improved efficacy in resource allocation but noting challenges in volatile environments (PMC (PubMed Central), 2022). Survey-based variants, such as Quadratic Voting Survey for Research (QVSR) deployed via the 2023 Civicbase platform, have elicited more nuanced public preferences in policy consultations, with pilots showing higher response quality than traditional polls (Bassetti et al., 2023). Ongoing research, including a 2025 thesis on quadratic surveys, continues to explore QV as a preference elicitation tool in empirical settings (Cheng et al., 2025).

Overall, these studies suggest QV enhances substantive legitimacy by better reflecting intensity, though results vary by context, with stronger performance in cooperative settings, where participants are inclined to reveal preferences sincerely, and less so in competitive or adversarial environments, where strategic behavior or collusion can undermine performance (Goeree and Zhang, 2017).

5.3 Ethical and Strategic Considerations: Fairness, Equity

Ethically, QV promotes fairness by allowing committed individuals to amplify their voice, potentially countering the "tyranny of the indifferent majority" and fostering civic engagement. It aligns with subjective value theory by treating preferences as relational and intensity-dependent, enabling outcomes that respect diverse stakes. However, one of the strongest and most recurring objections is the fact that equity concerns arise if credits are tied to real money, risking influence disparities based on wealth—though most implementations use equal virtual budgets to mitigate this (Posner and Weyl, 2018). Strategically, QV is resistant to certain manipulations due to its increasing costs, outperforming plurality in asymmetric information scenarios under some conditions. Yet, vulnerabilities persist, such as collusion among voters or under-expression by risk-averse participants. In common-interest models, QV's equilibria dominate those of simpler systems, but adversarial settings may require safeguards (Lalley and Weyl, 2018).

5.4 Implementation Challenges and Voter Experience

Practical deployment of QV faces hurdles in scalability and usability. Voter experience studies indicate initial confusion with quadratic math, necessitating education tools like intuitive interfaces (e.g., sliders showing marginal costs) (Bassetti et al., 2023). Implementation challenges include digital infrastructure for secure credit allocation, as seen in blockchain pilots (Gitcoin, 2023), and accessibility for non-tech-savvy populations (PMC (PubMed Central), 2022); a 2025 protocol for QV in liquid democracy exemplifies these challenges with scalable traffic and ZKP-based security, though blockchain volatility remains a hurdle (Kowalchuk et al., 2025). Platforms like the previously-cited Civicbase address these by simplifying QVSR for surveys, but broader electoral adoption requires addressing computational tractability and cultural adaptation.

While QV advances value-sensitive design, ethical and practical objections must be addressed for wider acceptance. One possible way forward is recognizing how its deterministic framework limits handling probabilistic or entangled preferences—gaps that quantum-inspired voting may address, as explored next.

6 Quantum Voting: Computational Recovery of Relational Aggregation

Efforts to recover preference intensity through expressive voting systems like QV address key limitations of classical aggregation, yet remain fundamentally rooted in reductive assumptions of modern democratic theory. While QV enables richer expression, it still treats ballots as independent, flattened data points, without capturing the relational dynamics and contextual interdependencies that shape collective decision-making—dynamics articulated by the School of Salamanca and earlier thinkers like Ramon Llull and Aristotle. Bridging this gap requires a framework that can formally encode such interdependencies into the aggregation process.

Quantum Voting (QVq) offers one such framework: it represents ballots as quantum states in a Hilbert space, enabling superposition, entanglement, and interference to model how preferences are interdependent and context-sensitive. In what follows, we examine key QVq models that demonstrate how quantum principles can be applied to voting, beginning with the foundational results in quantum social choice that established QVq as a viable theoretical path.

6.1 Quantum Social Choice and Arrow's Theorem: Structural Violation

Arrow's Impossibility Theorem proves that no deterministic aggregation mechanism can simultaneously satisfy Unanimity, Independence of Irrelevant Alternatives (IIA), and Non-Dictatorship when dealing with three or more alternatives. In a groundbreaking 2017 paper, Bao and Yunger Halpern proposed a quantum analog of Arrow's Theorem, constructing "Quantum Constitutions" as completely positive trace-preserving (CPTP) maps acting on joint quantum voter profiles. Their model showed that by encoding ballots in Hilbert spaces and allowing for entanglement and interference, it is possible to violate Arrow's theorem (Bao and Yunger Halpern, 2017). Bao and Yunger Halpern's Quantum Majority Rule (QMR) demonstrates that when voter preferences are represented as quantum states, aggregation need not collapse into dictatorship, even while respecting quantum analogs of Unanimity and IIA. Cycles in voter preferences, which classically induce aggregation paradoxes, are handled in the quantum setting via superposed and entangled states that probabilistically resolve inconsistencies without deterministic flattening. See Annex 2 for the formal quantum social choice framework, including CPTP map definitions for the quantum SWF and the encoding of ballots in Hilbert space.

Sun et al. further refined this framework by addressing apparent limitations in Bao and Yunger Halpern's definition of Quantum IIA (QIIA) (Sun et al., 2021). Their Quantum Condorcet Voting (QCV) mechanism introduced a stricter, context-sensitive formulation of QIIA and provided a detailed proof of Arrow's structural failure in quantum social choice. By redefining IIA to account for precise probabilistic correlations between ballot states, Sun et al. solidified the theoretical foundations of QVq as a legitimate avenue for bypassing classical impossibility results.

6.2 Mechanisms of Quantum Voting: Logical Operators and Ballot Interference

Building upon these foundational results, in 2025 Sun et al. propose a modular framework for quantum aggregation based on quantum logic operations (Sun et al., 2025). Central to this approach are Quantum Logical Veto (QLV) and Quantum Logical Nomination (QLN), primitive operations implemented via quantum gates (e.g., Toffoli gates) that allow ballots to interfere constructively or destructively. These primitives can be composed into scalable constructs such as Quantum Logical Majority Voting (QLMV) and Quantum Board Vote (QBV), which aggregate complex ballot structures while preserving the relational interdependencies among voter states. A striking feature of these models is their capacity to resolve classical aggregation paradoxes, such as the doctrinal paradox (Kornhauser and Sager, 1986), through interference patterns that align individual judgments with coherent collective outcomes. In impartial voting games, quantum aggregation mechanisms have been shown to outperform classical probabilistic methods, achieving fairer distributions of influence by dynamically adjusting to the entangled preferences of participants (Sun et al., 2021).

6.3 Conceptual Advantages: Relational Aggregation and Contextual Fairness

The core advantage of Quantum Voting lies in its ability to model relational aggregation. Whereas classical systems treat votes as independent inputs, QVq frameworks inherently recognize that voter preferences are often interdependent and context-sensitive. Entanglement allows for the encoding of civic interdependencies, reflecting how individual preferences are shaped by social ties, shared deliberations, and mutual obligations. Interference patterns, in turn, enable aggregation mechanisms to adapt to contextual shifts, dynamically recalibrating collective outcomes based on the evolving constellation of preferences. (See Figure 2) This relational logic challenges Arrow's IIA—not as a flaw, but as an intentional, Arrow-compliant redefinition of independence within a dynamic aggregation framework (see Annex 1-2). This view parallels Rovelli's relational interpretation of quantum mechanics, in which the properties of any system are defined only in relation to the systems with which it interacts (Rovelli, 2022, 1996).

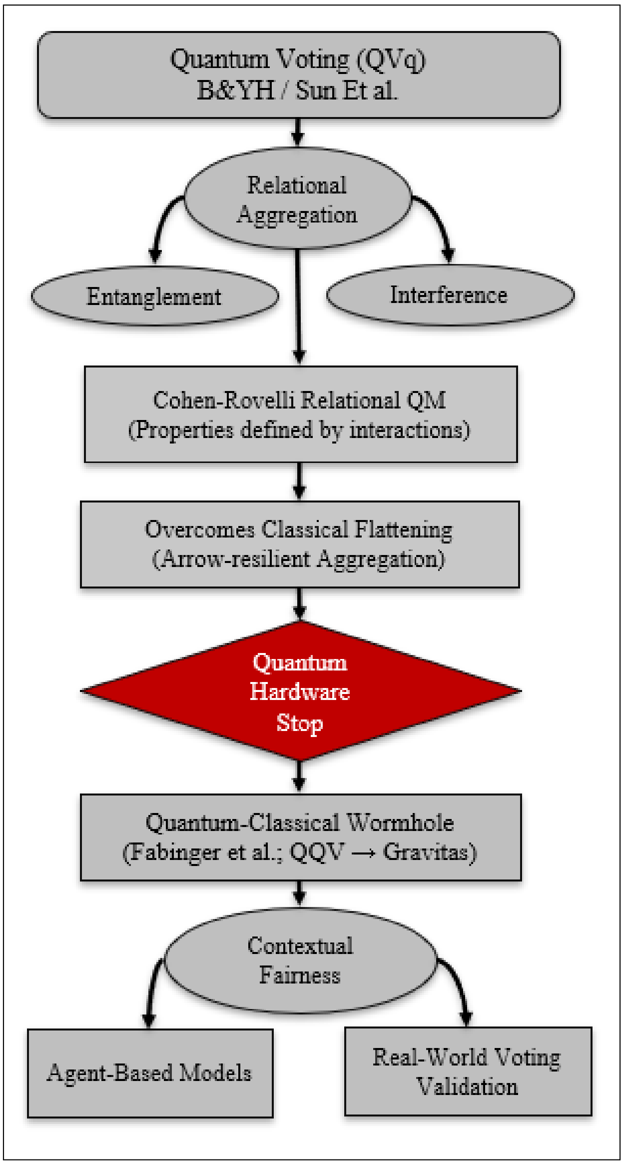

Figure 2: Derivation of QQV-Gravitas

Rather than treating voter preferences as isolated data points, Quantum Voting mechanisms allow contextual interdependencies to inform aggregation, ensuring that collective outcomes reflect the full relational fabric of preferences. This approach circumvents the flattening tendencies of classical systems, which often resolve complexity through arbitrary hierarchy or dictatorship. In QVq, deterministic aggregation is replaced by probabilistic unanimity, where coherent outcomes emerge through interference patterns that align individual judgments without sacrificing nuance. The result is a system that captures the intensity and relational dynamics of civic engagement, offering a richer, context-sensitive model of collective decision-making.

However, there is at least one major obstacle to QVq. While Bao and Yunger Halpern (Bao and Yunger Halpern, 2017) and Sun et al. (Sun et al., 2021, 2025) demonstrate QVq's theoretical strengths, their models assume access to quantum hardware to fully leverage superposition and entanglement in resolving preference cycles. This raises a critical question: Can these quantum advantages be realized through computational architectures that do not depend on full-scale quantum hardware?

Fabinger et al. posit a “wormhole” between quantum mechanics and quadratic financing, connecting the quadratic aspects of QM's Born rule (squaring amplitudes for probabilities) to the quadratic aggregation in classical QV (squaring the sum of square roots) (Fabinger et al., 2022). In Fabinger's Quantum Quartic Finance (Q4F), virtual processes act as citizens” contributing to public goods” (quantum events), with phased fourth roots amplifying outcomes probabilistically, mirroring QVq's interference while bridging to classical economics. This “wormhole” will be tested through quantum circuits that encode preference intensities as amplitudes, which are then squared to determine probabilities. Annex 2 includes the encoding of preference intensities and Born-rule-based probability extraction, with mapping to classical parameters planned in the completed integration pipeline.

The conceptual and mathematical foundation for this mechanism derives from Razo's (2014) Voluntary Taxation (VT) model, where voting power is quadratically weighted ($V = 1 + [\text{AVTR} \cdot x_i]^2$) based on a voter's average lifetime voluntary tax rate (Razo, 2014). In EC2, this logic provides a practical realization of the wormhole concept, implemented in a quantum circuit that translates directly into algorithms deployable within classical agent-based models (ABMs) for empirical testing of Arrow-resilient aggregation systems without requiring quantum hardware. While decision-making occurs in a classical, human context, the same aggregation logic can be executed within ABMs. This design ensures that key measurable parameters—such as civic commitment—can be represented consistently in both the quantum formalism and its ABM counterpart, enabling an interpretable mapping between the Annex 1–2 framework and the simulation environment. Concretely, Annex 2 defines the quantum SWF and its parameters, which in EC2 are intended to correspond to measurable variables such as civic commitment for integration with the ABM in the completed pipeline. This interpretability allows the ABMs to serve as realistic testbeds for quantum-inspired aggregation mechanisms, which can then be validated in real-world voting scenarios.

6.4 Challenges and Theoretical Frontiers

While full-scale Quantum Voting remains, for now, a theoretical construct—constrained by the technological challenges of scalable qubit architectures and interpretability of quantum states—the core principles of quantum aggregation are already being translated into classical computational frameworks. Through agent-based modeling and relational weighting mechanisms, the key advantages of quantum aggregation—contextual coherence, relational interdependencies, and Arrow-resilient aggregation—are being approximated and tested today (European City Squared Project, 2025).

Normative concerns also arise. The very features that grant QVq its expressiveness—contextuality, nondeterminism, relational encoding—may be perceived as opacity or arbitrariness in settings where democratic legitimacy depends on clear procedural fairness. Ensuring that QVq mechanisms remain interpretable, accountable, and resistant to technocratic elitism is a critical frontier for further research.

Additionally, foundational questions remain open: How should fairness be redefined in aggregation frameworks that inherently violate independence and determinism? Can new normative criteria—rooted in relational ethics and contextual coherence—replace classical axioms in evaluating democratic legitimacy? These challenges are not limitations but invitations—to rethink fairness as an emergent, relational property, and to design democratic systems where legitimacy is built not on rigid axioms, but on dynamically aligned collective valuations.

Operationalizing this vision requires translating quantum aggregation principles into tractable, interpretable computational frameworks. The EC2 project addresses this imperative by developing Gravitas, the project's classical, agent-based modeling implementation of quantum aggregation dynamics. Gravitas approximates relational quantum aggregation through a computational "wormhole," enabling empirical testing of Arrow-resilient democratic mechanisms in real-world contexts.

6.5 Quantum Voting as a Conceptual Laboratory

Quantum Voting should not be viewed as a near-term replacement for expressive methods like QV, but as a conceptual laboratory for experimenting with a quantum version of Quadratic Voting (Quantum Quadratic Voting, or QQV) and other aggregation dynamics that classical systems inherently cannot model. QQV encodes both preference intensity and reciprocal civic valuation within a unified quantum framework, enabling relational aggregation through superposition, entanglement, and interference.

In the EC2 project, these quantum aggregation principles are adapted for implementation within current computational limits. The European City Squared project is currently addressing this by developing Gravitas, a hybrid quantum-classical aggregation framework that operationalizes QQV's two-way subjective valuation through scalable agent-based simulations. Drawing on the relational dynamics of quantum aggregation and the "quadratic wormhole" proposed by Fabinger et al., Gravitas modulates voter influence not only by preference intensity but by demonstrated commitment to the common good—forming a bidirectional valuation loop that is both value-agnostic and locally adaptable.

Although this framework is highly flexible across cultural and institutional contexts, its foundations are firmly rooted in physical and mathematical principles, ensuring rigorous and transferable applicability across diverse democratic scenarios. Through attraction–repulsion dynamics, quadratic weighting, and boundary-condition optimization—inspired by Eliahu Cohen's work on a final boundary condition to the universe (Aharonov et al., 2017; Cohen, nd)—we wish to show how quantum principles can meet Arrow's fairness criteria while avoiding dictatorship.

Rather than imitating quantum aggregation through a purely classical simulation, we extract and apply its essential computational logic, such as superposition, entanglement, and the Born rule's treatment of amplitudes, to the design of voting mechanisms. In this framework, fairness arises naturally from relational structure and probabilistic coherence, rather than being imposed through rigid axiomatic rules.

Within this quantum-theoretical paradigm, we derive aggregation mechanisms that can be implemented in agent-based models for empirical testing. In EC2, the Gravitas model embodies this translation, using interpretable classical algorithms such as (1) updating each agent's influence weight quadratically from both preference intensity and reciprocal civic valuation, or (2) squaring the product of expressed preference and measured civic contribution, mirroring the Born rule, before normalizing across all agents.

Quantum Voting, in this sense, does not merely "bypass" Arrow's impossibility; it reframes aggregation as an emergent, relational process, where fairness is achieved through the dynamic alignment of individual and collective valuations, not through static axiomatic conditions. The computational "wormhole" connecting Quantum Voting, classical Quadratic Voting, and relational social dynamics provides a pathway for bringing these principles into agent-based simulations and real-world democratic experiments. By embedding these mechanisms within computationally tractable environments, Gravitas transforms Quantum Voting from a lofty theoretical construct into a practical design platform for structurally meeting Arrow's fairness constraints and, more importantly, creating resilient democracies that may be less prone to authoritarianism.

7 A Lost Lineage: Value and Voice in the European Tradition

The persistent challenges facing modern democracy—polarization, declining trust, strategic manipulation, rising authoritarianism—are often diagnosed as failures of implementation (Freedom House, 2023). Yet beneath these visible dysfunctions lies a deeper conceptual flaw: the assumption that political value is objective, evenly distributed, and exhaustively captured by numerical equality (Posner and Weyl, 2018). This assumption, embedded in the principle of "one person, one vote," stands in contrast to a largely forgotten intellectual tradition that treated value—whether political, moral, or economic—as inherently subjective, relational, and consent-based (Koselleck, 2004; Fortescue, 1997). The Enlightenment's emphasis on universal equality, while foundational to modern democracy, sidelined earlier frameworks, such as those of the School of Salamanca's Luis de Molina (Molina, 1614), which emphasized mutual consent and subjective valuation in governance (Nagel, 1999; Posner and Weyl, 2018). This section recovers these insights, arguing that their neglect exacerbates democratic vulnerabilities—such as strategic manipulation and authoritarian tendencies—that computational approaches like EC2 aim to address through relational valuation.

To address these vulnerabilities, revisiting this tradition illuminates conceptual blind spots in modern social choice theory and preference aggregation. While the political structures of medieval Europe were dominated by feudal hierarchies and rigid social orders, their leading thinkers often drew—directly or indirectly—on classical Greek traditions in which governance was grounded in subjective valuation, civic responsibility, and mutual recognition, as in Aristotle's conception of the polis as a reciprocal community aimed at the common good (Morrison, 2013; Celano, 2024). Though not institutionalized in their time, these principles resonate deeply with the computational design approaches we now seek to operationalize, offering an alternative lineage of democratic legitimacy grounded in reciprocal valuation rather than mathematical equality.

One of the earliest figures in this lineage is Ramon Llull, the 13th-century Catalan philosopher and logician, who devised the first algorithmic system for collective judgment. In his *Ars Electionis*, Llull proposed a method for making fair decisions through exhaustive pairwise comparisons, centuries before the Condorcet method formalized similar principles (Gilbert and Marijuan, 2001). Llull's framework was not democratic in the modern sense—it presupposed the wisdom of select judges rather than universal participation—but it was groundbreaking in its computational framing of fairness. He believed that legitimacy could emerge from structured reasoning processes, not merely from authority, laying conceptual foundations that now underlie computational social choice. In this sense, Llull's work anticipates key elements of EC2's approach: the idea that fairness in collective decision-making arises from relational, transparent, and systematic procedures rather than from rigid formulas that ignore human differences in commitment to the common good.

This lineage stands in stark contrast to the more prevalent tradition epitomized by Niccolò Machiavelli's *The Prince*, where governance is framed as an exercise in strategic control, manipulation, and the consolidation of power by rulers over subjects (Machiavelli, 1998). Machiavelli's realism, while influential in shaping modern political science, embodies a one-way valuation dynamic: the ruler evaluates and governs the populace, but the populace's valuation of the ruler—except in terms of pragmatic obedience—is largely irrelevant to legitimacy. The tradition under study, by contrast, envisions legitimacy as a two-way moral exchange, where rulers and citizens are bound by mutual recognition, consent, and civic responsibility. In response to the historical abuses of unchecked power, modern democracies embraced numerical equality as a safeguard against domination, but in doing so, they inadvertently erased the nuanced, relational dimensions of civic influence that legitimate governance requires. Rather than reducing governance to a calculus of power, this tradition grounds political legitimacy in relational ethics and subjective valuation, principles that have been sidelined as democratic systems increasingly equated fairness with numerical equality, overlooking the relational dynamics of civic interaction.

Several centuries after Llull, the School of Salamanca in 16th-century Spain advanced a moral and economic philosophy that would quietly shape the underpinnings of modern capitalism. Thinkers like Luis de Molina, Francisco de Vitoria, and Juan de Mariana articulated a subjective theory of value, asserting that the worth of goods—and, by extension, taxes and laws—stemmed from individual judgment and voluntary consent rather than from intrinsic properties, divine command, or labor alone (Chafuen, 2003). For them, value was not an objective quantity to be measured but a relational judgment arising from social and ethical contexts, shaped by mutual estimation and voluntary consent. This logic extended to governance: taxation without public consent was condemned as tyrannical, and laws derived their legitimacy from their alignment with the common good, as discerned through collective moral reasoning. Citizenship, in this framework, was not a mechanical aggregation of votes but a locus of reciprocal responsibility, where civic participation entailed obligations to the broader ethical and social order (Molina, 1593; Mariana, 1599).

This ethos of relational valuation found institutional expression in decision-making rules like the principle of *maior et senior pars*—the greater and wiser part (Tierney, 1998). In many early European governance systems, legitimacy required both numerical support (*maior pars*) and the endorsement of those recognized as morally or experientially qualified (*senior pars*). This was not mere elitism; it was a practical ethic, rooted in the belief that civic voice should reflect civic weight—whether measured in wisdom, judgment, or commitment to shared outcomes. Influence was not inherited or bought but accrued through demonstrable contribution to the collective welfare. Though imperfect and exclusionary by modern democratic standards, this model captured a critical truth: not all preferences carry equal civic weight, and governance mechanisms must account for the qualitative dimensions of civic commitment.

The Enlightenment, however, introduced a radical shift. The ideal of universal equality before the law, while emancipatory in many respects, flattened these older notions of qualified civic voice into arithmetical equality. Liberal contractarianism and the proceduralism of "one person, one vote" abandoned the subjective and moral basis of political value in favor of quantifiable fairness. This conceptual break in political theory paralleled a similar evolution in economic thought. The School of Salamanca's consent-based valuation was overtaken by frameworks like Adam Smith's labor-based measure of value (Smith, 1937), which sought to anchor exchange in comparative labor effort, and Karl Marx's objective labor theory of value (Marx, 1990), which treated labor time as the intrinsic determinant of a commodity's worth, apart from subjective preferences or negotiated consent. In democratic systems, this shift reduced citizens to interchangeable voting units, disregarding the variations in civic engagement, moral commitment, and contextual stakes that had been central, at least in theory, to earlier models of governance.

The result is a category error that persists to this day: political value is treated as an objective, divisible commodity—like slices of a pie—rather than as a subjective, relational construct akin to trust or meaning. This flattening of civic voice into numerical equality has stripped modern democracy of the moral texture that once underpinned its legitimacy in the older European tradition.

Yet this tradition has not vanished entirely. The Danish concept of *samfundssind*—"social-mindedness"—embodies a living example of how civic contribution and shared responsibility can shape democratic legitimacy. During crises like the COVID-19 pandemic, Danish citizens accepted restrictions and redistributive measures not because of coercion but because they saw themselves as stakeholders in a shared moral project (Lindholt et al., 2023). Trust in institutions remained high—not due to blind faith, but because of an ethic of civic alignment that permeates both formal governance structures and everyday social behavior. Similar principles appear in other cultures, such as the African philosophy of ubuntu ("I am because we are") (Metz, 2011), Indonesia's *gotong royong* ("mutual cooperation") (Bowen, 1986), and the Andean ideal of *buen vivir* ("good living" through communal harmony) (Gudynas, 2011), all of which embed legitimacy in reciprocal responsibility and shared well-being.

Samfundssind exemplifies the kind of cultural substrate theorized by the School of Salamanca: a society where legitimacy is not imposed from above but co-created through reciprocal valuation and mutual consent. What Denmark has preserved informally, the EC2 project now seeks to formalize computationally. The Gravitas model proposes a system where civic voice is weighted not by status or financial power, but by measurable indicators of freely-given civic contribution, participation, and commitment. Through agent-based simulations and real-world pilot testing—such as in the city of Aarhus—EC2 aims to encode this reciprocal valuation dynamic into scalable democratic mechanisms. In doing so, it seeks to recover the lost art of democratic design: a tradition where legitimacy arises not from equality alone but from equality plus the relational ethics of shared responsibility and civic weight.

8 Comparative Evaluation and Cross-Disciplinary Synthesis

Having traced the limitations of classical voting systems and explored innovative mechanisms such as QV and Quantum Voting (QVq), we now turn to a comparative evaluation of these paradigms. This section systematically contrasts classical, quadratic, and quantum-inspired aggregation models, assessing their capacity to reflect subjective valuation, mitigate strategic vulnerabilities, and adapt to contextual civic dynamics. Furthermore, we synthesize insights from ethics, computational theory, and mechanism design to chart a cross-disciplinary pathway for democratic innovation.

8.1 Classical, Quadratic, and Quantum Voting Systems Compared

Classical voting systems, including majoritarian and proportional representation models, operate on a foundational assumption of objective equality. Each voter's input is treated as an equal data point, irrespective of the intensity of preference or the individual's contribution to the collective good. While this procedural equality provides a clear and administratively simple framework, it systematically flattens subjective value, leading to undesirable outcomes that may misrepresent societal welfare, especially in contexts with heterogeneous civic commitment.

QV represents a landmark advancement in overcoming these limitations. By allowing voters to express preference intensity through a quadratic cost mechanism, QV bridges a critical gap between directionality and intensity of preferences. It offers a principled, tractable framework that balances fairness and expressiveness, significantly reducing strategic distortions inherent in classical systems. However, QV's deterministic structure—while essential for clarity and fairness—operates within specific informational assumptions, such as voter rationality and symmetric information. Additionally, the mechanism faces a foundational design tension: in market contexts, the quadratic cost is typically monetary, ensuring that participants internalize the weight of their expressed preferences. Yet, applying monetary costs in democratic settings raises profound equity concerns, prompting designs that allocate artificial vote credits instead (Posner and Weyl, 2018). While this adaptation preserves formal fairness, it also weakens the incentive-alignment that gives QV its allocative power, necessitating further innovations to reconcile expressive richness with equitable influence.

One way to address these tensions is to extend Quadratic Voting's foundational principles into frameworks that can inherently model relational dependencies and contextual interdependencies—domains where individual preferences are not isolated signals but dynamically entangled with the collective civic fabric. As societies (and, as discussed in Section 11, even artificial agents) grapple with increasingly complex and interdependent civic challenges, aggregation mechanisms must evolve to capture not only the strength of individual preferences but also the relational (and technological) contexts that shape them. Quantum Voting provides a framework for modeling these entanglements, extending QV's expressive logic into a domain where collective coherence emerges from dynamic interdependencies.

Quantum Voting builds on these insights by embedding relational aggregation directly into the decision-making process. By encoding preferences as quantum states, QVq captures the interdependencies and contextual entanglements that shape collective judgments. Unlike QV, which treats voter inputs as independent cost-signals, QVq models preferences as dynamically evolving elements within a connected civic fabric, allowing aggregation mechanisms to adjust in real-time to relational shifts. This relational and probabilistic framework enables QVq to navigate Arrow's Impossibility constraints (Bao and Yunger Halpern, 2017; Sun et al., 2021), offering a more adaptive and contextually nuanced model of collective valuation.

8.2 Assessment of Fairness, Strategic Vulnerabilities, and Intensity Sensitivity

In terms of fairness, classical systems excel procedurally but falter substantively. They guarantee equal formal participation but often fail to produce outcomes perceived as just or representative, particularly when preference intensities vary widely. Quadratic Voting substantially improves substantive fairness by enabling intensity expression, aligning influence with genuine preference strength while preserving fairness through cost-based constraints. However, the full realization of QV's fairness potential depends on thoughtful design choices that ensure informational symmetry and foster intuitive voter understanding of the quadratic mechanism (Wellings et al., 2024).

QVq redefines fairness through relational coherence. Rather than enforcing equality through rigid rules, it achieves fairness as an emergent property of interference patterns that align individual and collective valuations. This shifts the fairness criterion from input equality to output coherence, where legitimacy arises from the fair and contextual alignment of preferences. This relational approach also addresses the aggregation-influence trade-off empirically documented by Lorenz et al., where even mild social influence can undermine collective accuracy by compressing opinion diversity (Lorenz et al., 2011c)—a vulnerability that persists in classical and quadratic systems through mechanisms like tactical voting, collusion, and strategic token allocation (Lalley and Weyl, 2017). QVq, by embedding interdependencies, diffuses individual strategic incentives, though some—but not all—models can introduce challenges in transparency and interpretability. The QQV model being developed within the EC2 project, for example, prioritizes transparency and interpretability, ensuring that the relational coherence of aggregation processes remains intuitively accessible and auditable, even as it captures the dynamic interdependencies of civic preferences.

On preference intensity, classical systems remain blind, treating all inputs equally regardless of urgency or stake. QV directly encodes intensity through allocative budgets, providing a structured pathway for preference-weighted influence. QVq inherently captures intensity through amplitude encoding and interference dynamics, offering a fluid and contextually adaptive model where intensity is not a static input but an evolving relational state. The QQV model developed in the EC2 project extends this logic further by incorporating not only the strength of individual preferences but also the subjective valuation of the group toward the individual as an additional layer of information. This reciprocal valuation dynamic allows aggregation mechanisms to account for contextual relevance, enriching the informational substrate of collective decisions while preserving transparency and interpretability.

8.3 Underlying Assumptions About Value: Objective vs. Subjective

A critical axis of differentiation among these systems lies in their treatment of value. Classical systems implicitly assume value is objective, evenly distributed, and exhaustively captured by vote counts. Quadratic Voting acknowledges subjectivity in preference intensity but retains an objective, mechanical aggregation process. Quantum Voting fully embraces value as a subjective, relational construct, rejecting the notion of isolated voter preferences in favor of entangled civic identities. This ontological shift has profound implications. Classical systems prioritize procedural legitimacy; QVq shifts the foundation of legitimacy from rigid axioms to relational coherence, where fairness emerges through the dynamic alignment of preferences within their social context. This computational perspective revives a foundational civic ethic: that legitimacy and fairness emerge not from abstract formal equality, but from a relational process of mutual recognition grounded in dignity, freedom, and civic responsibility.

8.4 Cross-Disciplinary Synthesis: Ethics, Computation, and Mechanism Design

Integrating these insights demands a synthesis that transcends disciplinary silos. From ethics, we recover the principle that civic voice should reflect civic weight—an idea with deep roots in Greek civic virtue, Roman republican concepts like *auctoritas* and *gravitas*, and Jewish covenant traditions, where communal legitimacy arises through reciprocal obligations and mutual recognition of shared responsibility. This ethos was later articulated in the intellectual traditions of Llull and Salamanca, and continues to be culturally embodied in the Danish ethos of Samfundssind.

From computational theory, we derive the tools to model complex relational dynamics, translating philosophical principles into operational algorithms. Mechanism design provides the bridge, offering frameworks to construct aggregation systems that balance expressiveness, fairness, and strategic resistance.

The convergence of these fields in the EC2 project is not merely theoretical. By embedding relational aggregation dynamics into agent-based models and leveraging quantum-inspired computational logic, EC2 operationalizes the recovery of subjective value in democratic design. Gravitas exemplifies this synthesis: a computational mechanism that modulates civic influence based on measurable contribution and commitment, validated through simulations, real-world pilot deployments, and retrospective modeling of historical governance data to assess how civic contribution dynamics have influenced institutional resilience and societal cohesion over time. Annex 2 provides the quantum SWF function that we will later use as a basis to develop the simulation protocols, aggregation algorithms, and parameter calibration methods supporting these validations.

This cross-disciplinary approach reframes democratic innovation not as a rejection of established principles, but as their computational extension. It enables societies to adapt aggregation mechanisms to their unique civic landscapes, ensuring that democratic legitimacy evolves through the dynamic alignment of individual and collective valuations.

In the following sections, we explore how Gravitas, though currently theoretical, builds on real-world cultural models like Denmark's Samfundssind to provide a computational framework for reciprocal valuation—charting a pathway from conceptual design to empirical validation through simulations and pilot testing. To situate this framework within a broader class of self-organizing systems, we first draw parallels to principles observed in biological, physical, and quantum domains.

9 Active Inference and the Quadratic Dynamics of Two-Way Subjectivism

Similar dynamics appear across biological, physical, and quantum domains (Krakauer, 2024). In developmental biology, Michael Levin's research on bioelectrical signaling shows how multicellular organisms maintain target forms through distributed "pattern memories": local cells exchange electrical signals that adjust their states toward a coherent anatomical goal, enabling self-repair without central control (Levin, 2014, 2021). In physics, Adrian Bejan's Constructal Law describes how flow systems—whether rivers, lungs, or traffic networks—evolve architectures that minimize resistance and maximize access, progressively reorganizing to improve flow efficiency (Bejan, 1997, 2000). In quantum theory, the Two-State Vector Formalism (TSVF) describes quantum systems as evolving under the joint constraints of both past and future boundary conditions: the present state is shaped not only by its history but by a specified target end state (Aharonov et al., 2017).

Together, these perspectives converge on a design principle for democratic aggregation: encode collective goals as an explicit future boundary condition within a shared state space, structure influence flows to reduce friction, and enable local agents to adapt dynamically so that, even under perturbation, the system reorganizes toward fairness, coherence, and representativeness. While we recognize that the properties of complex systems typically emerge from bottom-up dynamics and evolutionary processes, our approach does not attempt to dictate outcomes from the top down. Instead, it specifies high-level boundary conditions and local interaction rules that guide self-organization—shaping the fitness landscape so that fairness, coherence, and resilience emerge organically from the system's own dynamics.

Building on this cross-domain principle, Active Inference offers a computational framework for operationalizing two-way subjective valuation as a dynamic, feedback-driven process. By treating democratic systems as adaptive agents that seek to minimize the "free energy" between their current state and a desired future civic state, Active Inference provides the mathematical machinery to implement the same goal-oriented, self-organizing behavior seen in biological pattern regulation, optimized flow systems, and TSVF-governed quantum processes.

A central innovation in this framework is the operationalization of two-way subjective valuation—the dynamic alignment between how individuals value outcomes and how society reciprocally values individuals based on their civic contributions, locally defined. This requires moving beyond static aggregation methods toward a continuously adaptive feedback system. Active Inference provides the formal structure for this process, modeling agents as minimizing a quadratic quantity—free energy—which quantifies the discrepancy between internal predictions and external observations (Friston, 2010). This minimization is inherently dynamic, involving attractive and repulsive forces: agents are drawn toward anticipated states that reduce uncertainty (attraction) and adjust away from prediction errors (repulsion) (Parr and Friston, 2019). Critically, this dynamic mirrors the attraction-repulsion mechanisms in bounded confidence models, where agents iteratively adjust their expressed valuations based on proximity thresholds in opinion space (Lorenz, 2007).

This relational dynamic also echoes Llull's combinatorial method, where coherence emerges from exhaustive pairwise comparisons ($O(n^2)$) (Colomer, 2013). Modern Large Language Models (LLMs) implement a parallel, non-local principle through quadratic (all-pairs) self-attention, enabling each token to interact directly with all others—except where masking removes specific links.[1] This capacity for non-local relational inference underlies the roles LLMs play in our architecture, from generating synthetic preference data to modeling reciprocal valuation dynamics. Annex 2 defines the quantum SWF, which will serve as the foundation for the proposed SWF–ABM–LLM integration described here.

In the proposed architecture, LLMs perform a triple role as part of the two-way subjective valuation loop. First, they generate synthetic, context-rich preference and deliberation data for initialization and scenario exploration in the agent-based environment. Second, when coupled to the quantum SWF pipeline detailed in Annexes 2, they pre-process and structure incoming preference data from ABM agents, ensuring compatibility with both classical and quantum aggregation stages. Third, in a proof-of-principle configuration, LLMs can interact dynamically with the evolving state of the simulation through the quantum–classical "wormhole" interface, modulating preferences in real time via bidirectional feedback between quantum aggregation logic and

---

[1] In transformers, "quadratic" denotes the $O(n^2)$ scaling of self-attention, where each token attends to all others. In the EC2 framework, this reflects a deeper structural law: across domains—from Llull's art to Newton's inverse-square laws, quadratic terms in physics, Born-rule squaring, and gravitas-weighted aggregation—quadratic structure encodes global coupling. Such coupling enables coherence and stable attractors, collapsing high-dimensional potentials into constrained, interpretable outcomes, making self-attention conceptually akin to our aggregation model's quadratic weighting.

the ABM. This integration allows the quantum model to be parameterized in terms of interpretable classical variables facilitating meaningful mapping between the quantum formalism and the agent-based simulation. If this bidirectional link is realized, the first two roles converge into a continuous adaptive feedback loop, allowing LLMs to function not only as data generators but as embedded adaptive agents. This operational linkage grounds the LLM component directly in the formal computational architecture of Annexes 1 and 2, providing a reproducible pathway for empirical testing of the quantum–classical aggregation mechanisms proposed in Sections 5–6.

In this sense, LLMs instantiate the continuous feedback loop between individual valuation (expressed preferences) and societal recognition (assigned influence), ensuring that aggregation mechanisms reflect both sides of the valuation process in real-time—a process formally analogous to minimizing variational free energy, where agents and systems iteratively align internal predictions with external feedback to reduce uncertainty and optimize relational coherence (Friston, 2010).

At the core of this architecture, Active Inference drives a continuous feedback loop, where agents constantly update their preferences to minimize prediction errors about their influence in the collective. At the system level, the aggregation mechanism—simulated in the agent-based model—adjusts societal expectations in response, recalibrating how much influence and recognition each agent receives as the relational dynamics evolve.

In QQV, the amplitude assigned to each voter–option pair encodes both the voter's own preference intensity and the system's reciprocal valuation of that voter's contribution (their "civic gravitas"). Many quantum models use quadratic Hamiltonians, and all obey the Born rule, which converts amplitudes into probabilities by squaring them. By squaring the combined (preference × contribution) amplitude, QQV embeds both individual and societal valuations in the aggregation process, creating a "wormhole" that bridges this quantum aggregation logic to its classical approximation in agent-based models, as described by EC2 consortium partners Michal Fabinger and Glen Weyl (Fabinger et al., 2022).

By embedding Active Inference as the system's dynamic engine, we bring together Llull's combinatorial decision logic, LLM-driven relational inference, Quadratic Voting mechanisms, and quantum aggregation principles into a unified computational framework. This not only makes two-way subjective valuation practically computable but also connects it to a deeper lineage where relational coherence and preference collapse are fundamental organizing principles—in cognition, computation, and the physical world (European City Squared Consortium, 2025).

10 Danish Samfundssind: From Computable Ethics to Scalable Democracy

The Gravitas model represents a structured attempt to encode subjective valuation into democratic aggregation processes, operationalizing civic influence as a function of measurable contribution and contextual relevance. Unlike classical voting systems that treat all inputs as equal regardless of commitment or stake, Gravitas introduces a relational framework where civic weight dynamically modulates voice, ensuring that legitimacy arises through reciprocal valuation between individuals and the collective. This architecture, rooted in quantum computational principles, is not prescriptive; it is designed as a value-agnostic mechanism that societies can calibrate to reflect their unique civic norms and expectations.

Denmark's concept of *Samfundssind* offers a living empirical foundation for Gravitas. Rooted in cultural norms of social-mindedness, mutual responsibility, and civic trust, *Samfundssind* exemplifies how societies can achieve cohesive governance through informal reciprocal valuation, without needing formal mechanisms to enforce it. However, Denmark's civic success is not merely a function of cultural norms—it is also structurally supported by its relative homogeneity and by a relatively narrow distribution of civic commitment across its population. The gap between the most and least civically engaged citizens in Denmark is modest, allowing traditional one-person-one-vote systems to produce outcomes that are broadly perceived as fair and effective.

In such homogenous civic landscapes, the need for dynamic aggregation adjustments is minimal, and yet, a mechanism like Gravitas could further enhance efficiency and fairness by fine-tuning influence based on measurable contribution. For instance, Gravitas could allow those less engaged to bear a lighter share of collective burdens, while those more deeply committed to *Samfundssind* assume a proportionally greater role—optimizing outcomes without disrupting the underlying social contract.

In more heterogeneous societies, where the distribution of civic commitment is wider, Gravitas becomes not just an enhancement but a critical corrective tool. By dynamically weighting civic voice according to demonstrated contribution and contextual relevance, Gravitas can compensate for the disparities that cause one-person-one-vote systems to falter, steering collective outcomes toward the cohesion and resilience exemplified by the Danish model. In this sense, Gravitas provides a scalable mechanism for approximating Denmark's civic coherence in contexts where cultural homogeneity cannot be assumed.

The EC2 project seeks to formalize this dynamic through Gravitas, translating the informal relational coherence of *Samfundssind* into a computationally tractable model that can be tested, iterated, and adapted across diverse governance contexts. The objective is not to replicate Danish culture, but to extract and encode the aggregation logic that underpins its civic resilience, enabling societies to design democratic mechanisms that reflect both subjective valuation and contextual civic dynamics.

Aarhus, Denmark, serves as a strategic testbed for this endeavor. Through agent-based simulations and small-scale, real-world piloting and surveys, EC2 will evaluate how various algorithms—from 1p1v to QV to QQV—perform in settings where civic-mindedness is already deeply ingrained, providing a baseline for assessing the model's capacity to preserve and enhance relational legitimacy. From there, the project will explore how Gravitas can be adapted to contexts with varying degrees of civic cohesion, testing its scalability and adaptability as a universal democratic infrastructure.

This pathway from Denmark to the world is not merely a cultural export but a methodological exploration of how computable ethics can bridge the gap between procedural fairness and substantive legitimacy. By grounding Gravitas in empirical realities while preserving its flexibility as a value-sensitive design framework, EC2 aims to create scalable democratic mechanisms that are both transparent and resilient, capable of fostering legitimacy through the dynamic alignment of individual and collective valuations.

11 Emergent Altruism — Reconciling Political-Economic Divides

The tension between individual autonomy and collective welfare—a foundational dilemma in political philosophy—has shaped governance debates across civilizations for millennia. From ancient notions of civic virtue and republicanism in Greece and Rome to Confucian ideals of social harmony and Jewish covenant traditions of mutual responsibility, societies have long grappled with how to balance personal initiative with obligations to the common good. This enduring philosophical tension has crystallized into the more familiar ideological divide between "left" (egalitarian, redistributive) and "right" (market-driven, meritocratic) frameworks, which have structured political economy and tax debates for the past several centuries (Razo, 2014).

On one side, institutional and Keynesian traditions have advocated for centralized mechanisms to ensure equity and safeguard public welfare. On the other, neoclassical, Austrian, and Chicago School economists have emphasized market efficiency, individual initiative, and minimal state intervention. Both traditions respond to legitimate challenges: the former seeks to correct systemic inequities, while the latter aims to harness decentralized information and incentivize productive contribution. Yet, systems built on either pole have often oscillated between extremes, struggling to reconcile fairness with dynamism, or efficiency with legitimacy.

This enduring tension between individual and collective valuation has its roots in a long-standing philosophical shift: from relational models of civic influence—where recognition and responsibility were mutually reinforcing—to procedural egalitarianism, which treats political voice as an abstract uniform entitlement. By severing the dynamic feedback loop between civic contribution and political influence, modern systems risk eroding legitimacy and social coherence.

The concept of emergent altruism, not to be confused with effective altruism, offers a pathway to transcend this dichotomy—not by compromising between opposing visions, but by rethinking how civic value is aggregated and rewarded. Emergent altruism, a term coined by the EC2 project, describes a dynamic wherein individuals, acting within voluntary competitive frameworks, are incentivized to contribute to the common good because doing so enhances their influence and standing within the collective. This phenomenon, explored in recent economic contract theory models of mutualism, demonstrates how cooperative behavior can arise endogenously from systems that align individual benefit with collective welfare through structured feedback mechanisms (Weyl et al., 2010). Drawing on Llull's early computational logic and Salamanca's relational ethics, this approach revives the idea that legitimacy stems from mutual recognition, where civic weight reflects demonstrated commitment rather than inherited or arbitrary status.

Emergent altruism in QQV arises from the interplay between individual expression and collective reciprocity. By linking influence to measurable civic engagement, the system turns contribution into a shared signal of trust, encouraging cooperative norms to develop without prescriptive control. This dynamic shapes long-term expectations more than immediate vote tallies, rewarding sustained public-minded behavior in ways that are visible, auditable, and resistant to manipulation. In doing so, QQV aligns personal incentives with collective benefit, making altruism an emergent property of the aggregation process rather than a top-down directive.

This approach reframes political competition. Instead of vying for influence through wealth accumulation or identity-based majoritarianism, citizens, institutions, and even artificial agents, compete through their contributions to collective welfare—be it through voluntary taxation, civic participation, or public service. Influence is earned, not assumed, and societal outcomes emerge from a dynamic alignment of individual contributions with collective valuation. In this sense, QQV offers a computational instantiation of emergent altruism, operationalizing principles long theorized but never structurally encoded in democratic mechanisms.

Preliminary NetLogo-based simulations suggest that even simple weighting of influence by civic contributions (e.g., community service hours) can increase perceived fairness and reduce polarization in diverse agent groups, supporting the viability of emergent altruism dynamics for further empirical testing.

The implications for reconciling ideological divides are profound. For proponents of market-based systems, QQV retains the adaptive efficiency of competitive dynamics, ensuring that influence flows toward those who contribute most. For advocates of social equity, it provides a structured mechanism to recognize and reward civic responsibility, preventing the capture of power by disengaged elites or free riders. Crucially, QQV is not a prescriptive ideology; it is a value-agnostic infrastructure that societies can calibrate to reflect their own civic norms, balancing contribution-based influence with foundational principles of equality and fairness.

Emergent altruism also provides a stabilizing force against the systemic pathologies afflicting modern democracies—polarization, strategic manipulation, declining trust, and rising authoritarianism. By embedding a continuous feedback loop between civic contribution and political influence, QQV fosters a culture of reciprocal responsibility, where legitimacy is co-created through transparent and interpretable aggregation dynamics. This computational architecture does not abolish traditional democratic institutions; it enhances them, offering a relational layer of governance that aligns influence with stake, commitment, and societal benefit.

Moreover, by positioning this emergent dynamic within agent-based models and computational social choice frameworks, EC2 provides a testable platform for institutional experimentation. Controlled simulations and pilot deployments allow groups to explore how different calibration strategies affect social cohesion, governance efficiency, and perceived fairness—without the risks and costs of real-world policy experimentation. This iterative, data-driven approach to democratic design marks a significant departure from the static, one-size-fits-all models that have dominated political theory since the Enlightenment.

In this way, QQV offers a quantum computational reconciliation between competing political-economic schools of thought. It retains the adaptive, decentralized virtues of competitive systems while embedding safeguards for equity, civic trust, and reciprocal valuation. By grounding governance legitimacy in dynamic, contribution-sensitive feedback mechanisms, this emergent altruism provides a conceptual and technical bridge across ideological divides, re-aligning democratic design with the complex, interdependent realities of 21st-century societies.

12 Future Research Directions

The history of social and political innovation is marked by both visionary breakthroughs and costly missteps. Past experimentation with governmental systems—when pursued without rigorous empirical grounding or sensitivity to societal complexities—has often resulted in unintended consequences, undermining trust and legitimacy. Recognizing this, the EC2 project approaches the operationalization of QQV and related aggregation mechanisms with a deliberate emphasis on caution, empirical validation, and adaptive scalability.

Scaling Gravitas through Agent-Based Simulations: Beyond their utility as a validation tool, agent-based simulations enable rapid, cost-effective, and risk-free experimentation with aggregation mechanisms that would be prohibitively slow, expensive, or too socially disruptive to test in real-world settings. By iterating on simulated civic environments, EC2 can explore a wide array of design variations, stress-test their robustness under diverse conditions, and refine voting algorithms in a sandbox while safeguarding societal trust and institutional stability.

Quantum Models of Voter Behavior and Aggregation Dynamics: Building upon the theoretical foundations of Quantum Voting, future research will further experiment with quantum circuit implementations of aggregation operators such as Quantum Majority Rule (QMR), Quantum Condorcet Voting (QCV), and Quantum Quadratic Voting (QQV). Future models will continue to explore if and how entanglement and interference patterns structurally resolve preference cycles and strategic manipulation. Additionally, research will focus on computational "wormholes" that translate quantum aggregation logic into scalable classical approximations, ensuring that theoretical advances remain computationally tractable and interpretable before any application in real-world governance scenarios.

Empirical Testing of 1p1v, Quadratic Voting, and Quantum Quadratic Voting: Empirical testing is a cornerstone of responsible innovation. Comparative simulations and small-scale field experiments or surveys will be designed to evaluate the performance of classical 1p1v, QV, and QVq systems in controlled contexts.

The goal is not to promote wholesale replacement of existing systems but to assess the specific conditions under which alternative aggregation mechanisms can enhance perceived fairness, civic engagement, and outcome legitimacy. Initial pilots, such as those planned in Aarhus and Basel, will provide empirical data while ensuring that interventions remain locally grounded and reversible.

Ethical Safeguards and Cultural Calibration: Ensuring that our research enhances inclusivity, transparency, and legitimacy requires robust ethical safeguards and cultural calibration protocols. Research will focus on designing adaptable weighting mechanisms that can and should be aligned with diverse cultural norms and societal values, while avoiding biases. Special emphasis will be placed on transparency and interpretability, ensuring that aggregation outcomes remain understandable and accountable to all stakeholders. These safeguards will be developed in parallel with technical testing, forming an iterative feedback loop that continuously aligns mechanism design with societal expectations.

Integration with Participatory and Deliberative Systems: Recognizing that no social choice mechanism can substitute for the depth of collective reasoning inherent in participatory and deliberative processes, future research will explore how Gravitas can complement these democratic practices. In the medium and long run, we envision hybrid models where civic contribution and deliberative participation dynamically influence aggregation weights, ensuring that computational efficiency is balanced with the deliberative depth and contextual nuance necessary for legitimate decision-making.

AI Governance and Artificial Agent Aggregation: As AI systems become increasingly autonomous and interconnected, the challenge of aggregating machine-generated preferences in ways that align with human values becomes critical. Gravitas offers a physics-and-ethics-informed architecture to address this need. Future research may explore how this framework can be applied to AI governance scenarios, ensuring that autonomous agents operate within aggregation logic that remains transparent, interpretable, and aligned with the collective good.

Practitioner Innovations and Applied Experiments: Ongoing efforts, such as Kenrick Nelson's 2025 Cardano simulations exploring Quadratic Voting in AI governance, represent valuable applied experiments that inform the EC2 research agenda (Kovalchuk et al., 2025). In addition, there is a growing body of research, notably from Glen Weyl's RadicalxChange community, which explores QV with reputation mechanisms (e.g., vote-escrowed tokens, soulbound tokens, or decay-based reputation scores) to enhance fairness and resist sybil attacks and whale dominance in blockchain and DAO settings (Buterin et al., 2018). Notable references include:

- Weyl and Buterin's theoretical foundation for reputation-based QV (Buterin et al., 2018).
- Nelson KP et al. Quadratic Voting simulations (Nelson et al., 2024).
- Kowalchuk L et al. Universally Composable On-Chain Quadratic Voting (Kovalchuk et al., 2025).

These initiatives provide empirical insights that will inform the iterative development of the EC2 platform, ensuring its adaptability and relevance across governance landscapes. These applied experiments will be engaged with a critical perspective, recognizing the need for broader societal validation before any large-scale deployment.

Adoption Pathways and Social Uptake: Adoption of QQV and related mechanisms is not merely a political or administrative hurdle; it is a complex research challenge involving behavioral dynamics, institutional design, and societal trust-building. Future research will investigate the factors that influence the acceptance and integration of new aggregation logics within existing governance structures, including resistance points, incentive structures, and stakeholder engagement strategies. Adoption pathways will be explored through incremental, reversible piloting that prioritizes local legitimacy and iterative feedback.

A core component of the adoption strategy involves active interaction and knowledge-sharing with other Horizon Europe democracy projects. Collaborative efforts will focus on harmonizing methodologies, sharing empirical findings, and co-developing best practices for adaptive democratic design. This networked approach will not only accelerate EC2 development and validation but also ensure that adoption strategies are informed by a diverse range of democratic innovation initiatives across Europe. A number of these planned research activities rely directly on the formal and computational frameworks already set out in Annexes 1 and 2.

Technical Foundations in Annexes 1–2

Annexes 1 and 2 provide the formal and computational underpinnings for the concepts developed in this review. Annex 1 /2 contains the classical / quantum social choice frameworks, including the formal derivation of the quadratic cost–utility relationship, the definition of the quantum SWF function, and the mathematical structures linking vote costs to welfare maximization. While Annex 2 does not yet include the full SWF–ABM integration pipeline, it provides the base for developing the simulation protocols, aggregation algorithms, and parameter calibration methods discussed in the main text. Together, these Annexes transform the theoretical proposals in Sections 5, 6, and 8–10 into a reproducible technical framework, ensuring that the review's arguments are grounded in rigorously defined models and are positioned for empirical testing in both classical and quantum settings.

13 Conclusion

Modern democratic systems flatten political value into formal equality, overlooking variations in preference intensity and civic contribution. This oversight, rooted in Enlightenment egalitarianism, has constrained aggregation mechanisms to procedural fairness at the expense of substantive legitimacy and optimized social outcomes. Recovering earlier traditions—from Llull's algorithmic reasoning to the School of Salamanca's consent-based valuation—offers a pathway to reintegrate subjective value into democratic design.

There is no reason to believe that our current democratic systems represent the final stage of institutional evolution—and many reasons to believe they do not. The escalating threats of nuclear conflict, rising authoritarianism, and deepening societal polarization all point to a critical need for democratic innovation. Far from being static, democracy must continuously refine its mechanisms to meet the complex and evolving demands of governance in the 21st century.

Building on innovations like Quadratic Voting, the EC2 project advances this trajectory through Quantum Quadratic Voting: a computational framework that, among other options, modulates civic influence based on measurable contribution and contextual relevance. Rather than proposing immediate adoption, EC2 emphasizes rigorous testing through agent-based simulations, enabling rapid, risk-free experimentation to refine aggregation dynamics before any real-world deployment. Controlled pilots in Aarhus and Switzerland will provide initial empirical grounding, while broader adoption strategies will be explored not only as a research frontier, but also as grounds for the adoption of innovative democratic practices among policymakers and civil society in the arena of electoral politics and electoral policymaking.

By embedding subjective valuation into scalable, transparent computational models, EC2 seeks to contribute to the evolution of our understanding of democracy through dynamic, reciprocal civic alignment—bridging historical insight with responsible innovation for the complexities of 21st-century governance.

Annex 1 Classical social choice theory: ranked-voting and Arrow's theorem

Annex 1.1 Notations

Notion	Definition	Description
Set of voters	$V = \{v_1, \dots, v_n\}$	n voters
Coalition	$C \subseteq V$	Any subset of the voters set.
Set of alternatives	$A = \{a_1, \dots, a_m\}$	m alternatives
Binary relations:	$x, y \in A$	
Weak	$x \succsim y = (x, y)$	x is *at least as good as* y
Strict	$x \succ y$	x is strictly preferred to y
Set of pairs	$A \times A = \{(x, y) \mid x, y \in A\}$	All possible pairs
Subsets:		
Complete	$s_c \in A \times A : \forall x, y \in A,\ \ (x, y) \in s_c \vee (y, x) \in s_c$	All pair relations appear in s_c
Transitive	$s_t \in A \times A : \forall x, y, z \in A,\ \ (x, y) \in s_t \wedge (y, z) \in s_t \Rightarrow (x, z) \in s_t$	$x \succsim y$ and $y \succsim z$ imply $x \succsim z$
Reflexive	$s_r \in A \times A : \forall x, y \in A, (x, y) \in s_r \wedge (y, x) \in s_r \Rightarrow x \sim y$	Allows ties
Irreflexive	$s_i \in A \times A : \forall x, y \in A, (x, y) \in s_i \Rightarrow (y, x) \notin s_i$	No ties
Orders:		
Partial weak	$r(A) = \{s \in A \times A \mid s \text{ is transitive and reflexive}\}$	May include ties. Not complete.
Partial linear	$l(A) = \{s \in A \times A \mid s \text{ is transitive and irreflexive}\}$	No ties. Not complete.
Weak	$R(A) = \{s \in A \times A \mid s \text{ is complete, transitive and reflexive}\}$	May include ties
Linear	$L(A) = \{s \in A \times A \mid s \text{ is complete, transitive and irreflexive}\}$	No ties
Set of weak orders	$\mathbf{R}(A) = \{R(A)\}$	All weak orders
Set of linear orders	$\mathbf{L}(A) = \{L(A)\}$	All linear orders
Voter i's preference	$L_i = \big(a_{i(max)} \succ \cdots \succ a_{i(min)}\big) \in \mathbf{L}(A), i \in V$	Linear order
Profile	$P = (L_1, \dots, L_n) \in \mathbf{L}(A)^n$	All voters' preferences in an election
SWF	$f(P) : \mathbf{L}(A)^n \to R(A)$	The social preference is a weak order, when the voters' ones are weak.
Set of voters in P unanimous on a certain pair	$N^P_{a \succsim b} = \{i \in V : a \succsim_i b\} \subseteq V$ $N^P_{a \succsim b} \cap N^P_{b \succsim a} = \emptyset,\ \ N^P_{a \succsim b} \cup N^P_{b \succsim a} = V$	All the voters in profile P who, for example, rank a above b.
Weakly decisive coalition	$a \succsim_{\text{soc}} \vee b \succsim_{\text{soc}} a \Rightarrow N^P_{a \succ b} = W^P_{a \succ b} \vee N^P_{b \succ a} = W^P_{b \succ a}$	Either $N^P_{a \succ b}$ or $N^P_{b \succ a}$ is a weakly decisive coalition, since society either prefers a to b, or the opposite.

Table 1: Classical ranked-voting notations

Annex 1.2 Arrow's properties

We would like to briefly introduce the ranked-voting scheme's properties according to Arrow:

1. Transitivity: Society's preferences $a \succsim_{\text{soc}} b$ and $b \succsim_{\text{soc}} c$ imply $a \succsim_{\text{soc}} c$.
2. Unanimity: If every voter prefers a to b, then society's preference should be the same: $\forall i \in V, a \succsim_i b \Rightarrow a \succsim_{\text{soc}} b$ for any $a, b \in A$.
3. Independence of irrelevant alternatives (IIA): The relative ranking of any two candidates, say a and b, should not be influenced by the ranking of any other candidate. Given two preference profiles (different elections), $P = (L_1, ..., L_n)$ and $P' = (L'_1, ..., L'_n)$, if all the individual preferences regarding any two alternatives coincide in the profiles P and P', then the social preferences coincide as well. $\forall i \in V, r_i^{ab} = r'^{ab}_i \Rightarrow r^{ab}_{\text{soc}} = r'^{ab}_{\text{soc}}$, where $r \in \{\succsim, \precsim\}$.
4. Dictatorship: Society prefers a to b iff some voter i prefers a to b: $\exists i : a \succsim_i b \;\Leftrightarrow\; a \succsim_{\text{soc}} b$.

Annex 1.3 Arrow's theorem - Proof by a decisive coalition

According to Arrow's impossibility theorem, every SWF with three alternatives or more that satisfies transitivity, unanimity, and IIA, is inevitably a dictatorship.

Claim 1: *Contagion Lemma* or *Field Expansion Lemma*

A weakly decisive coalition for a versus b is decisive for all the pairs.

Proof:

Consider a general profile P with a weakly decisive coalition for a versus b, $C = W^P_{a \succsim b} = N^P_{a \succsim b}$, namely:

$$a \succsim_i b, \quad \forall i \in C,$$

and

$$b \succsim_i a, \quad \forall i \in N \setminus C = N^P_{b \succsim a}.$$

Using IIA, we can modify the profile P to P', maintaining the social preference, $a \succsim'_{\text{soc}} b$, such that C is also unanimous about:

$$x \succsim'_i a \succsim'_i b \succsim'_i y, \quad \forall i \in C = W^{P'}_{a \succsim b},$$

and

$$b \succsim'_i y, \quad x \succsim'_i a, \quad \forall i \in N \setminus C = N^{P'}_{b \succsim a} = N^P_{b \succsim a}.$$

By definition, $b \succsim_j a$, $j \in N \setminus C = N^{P'}_{b \succsim a} = N^P_{b \succsim a}$. From the unanimity of SWF we have

$$x \succsim'_{\text{soc}} a, \quad b \succsim'_{\text{soc}} y,$$

but, since, due to the weak decisiveness of $W^{P'}_{a \succsim b}$ we have $a \succsim'_{\text{soc}} b$, transitivity assures that $x \succsim'_{\text{soc}} y$. That means that the weakly decisive coalition $W^{P'}_{a \succsim b}$, in the the modified profile P', is (not weakly) decisive for (x, y), namely, $C = D^{P'}_{x \succsim y}$, since we didn't assume anything regarding the preferences concerning (x, y) outside the coalition, $|D^{P'}_{x \succsim y}| \leq |W^{P'}_{x \succsim y}|$. According to IIA we, can change P' back to P without changing the preferences between a and b, and x and y maintaining $a \succsim_{\text{soc}} b$, and $x \succsim_{\text{soc}} y$, making the coalition to be decisive for x and y, in addition to being weakly decisive for a and b,

$$C = W^P_{a \succsim b} = D^P_{x \succsim y}, \quad \forall x, y \in A \setminus \{a, b\}, \quad \forall P \in \mathbf{L}(A)^n,$$

as claimed.

Claim 2: ***Splitting Lemma*** **or** ***Group Contraction Lemma***

If a coalition is decisive, then it has a proper subset that is also decisive. By repetition, a single individual coalition is also decisive, making it a dictator.

Proof:

Consider a profile P with a decisive coalition C.

Modify P to P' such that $C_1 \cup C_2 = C$ and $C_1 \cap C_2 = \emptyset$ where

- $\forall i \in C_1 = N^{P'}_{a \succsim_i c} : a \succsim_i b \succsim_i c$
- $\forall i \in C_2 = N^{P'}_{b \succsim a} : b \succsim_i c \succsim_i a$
- $\forall i \in N \setminus C = N^{P'}_{c \succsim b} : c \succsim_i a \succsim_i b$

Notice that $C = N^{P'}_{b \succsim c} = W^{P'}_{b \succsim c} \Rightarrow b \succsim_{\text{soc}} c$.

Case 1: If $a \succsim_{\text{soc}} c$, hence $C_1 = W^{P'}_{a \succsim c}$, and hence decisive for all pairs $\forall P' \in \mathbf{L}(A)^n$.

Case 2: If $c \succsim_{\text{soc}} a$, by transitivity $b \succsim_{\text{soc}} a$, $C_2 = W^{P'}_{b \succsim a}$, and hence decisive for all pairs $\forall P' \in \mathbf{L}(A)^n$.

By repetition of the *Splitting Lemma* steps, one can show that there exists a single-individual decisive coalition - a dictator, as claimed.

Annex 1.4 Quadratic Voting

Consider candidates $a_1, \dots, a_m$ and voters $i = 1, \dots, n$. Voter i has cardinal values $\theta_i = (\theta_{i,1}, \dots, \theta_{i,m})^\top$ for each candidate winning (defined up to an additive constant per voter). Under quadratic voting, voter i purchases a *vote vector* $v_i = (v_{i,1}, \dots, v_{i,m})^\top \in \mathbb{R}^m$ at cost

$$c_i(v_i) = \kappa \|v_i\|_2^2 = \kappa \sum_{j=1}^{m} v_{i,j}^2, \qquad \kappa > 0.$$

Aggregate scores are $S := \sum_{i=1}^{n} v_i \in \mathbb{R}^m$. The winner is chosen by a smooth, symmetric choice rule $P(S) = (P_1(S), \dots, P_m(S))$ (e.g., a softmax), so that $P_j(S)$ is the probability that a_j wins and $\sum_j P_j(S) = 1$. Each voter chooses v_i to maximize expected utility

$$U_i(v_i) = \theta_i^\top P(S_{-i} + v_i) - \kappa \|v_i\|_2^2, \qquad S_{-i} := \sum_{k \neq i} v_k.$$

Linearization near ties. Elections are decided where the scores are close, $S \approx s\mathbf{1}$ with $\mathbf{1} = (1, \dots, 1)^\top$. A first-order expansion gives $P(S) \approx P(s\mathbf{1}) + J(s\mathbf{1})(S - s\mathbf{1})$, where

$$J(s\mathbf{1}) = \alpha\,\Pi, \qquad \Pi := I - \frac{1}{m}\mathbf{1}\mathbf{1}^\top, \qquad \alpha > 0.$$

Here Π is the orthogonal projector onto the zero-sum subspace (it subtracts the mean across candidates).

Individual optimality and aggregation. With this linearization, the first-order condition for U_i is

$$J(s\mathbf{1})^\top \theta_i = 2\kappa v_i^* \implies v_i^* = \frac{\alpha}{2\kappa}\,\Pi\theta_i.$$

Summing across voters,

$$S^* = \sum_i v_i^* = \frac{\alpha}{2\kappa}\,\Pi\Theta, \qquad \Theta := \sum_{i=1}^{n} \theta_i.$$

Since Π subtracts the same constant from every coordinate, it preserves argmax:

$$\arg\max_j S_j^* = \arg\max_j \Theta_j.$$

Thus, quadratic voting makes the chosen candidate coincide (approximately, in the decisive region) with the *utilitarian* maximizer $\arg\max_j \sum_i \theta_{i,j}$.

Why quadratic. If costs were $c_i(v_i) = \kappa \|v_i\|_p^p$ with $p \neq 2$, the optimal v_i would be a nonlinear transform of θ_i, so $\sum_i v_i$ would not align proportionally with $\sum_i \theta_i$. The quadratic case $p = 2$ is the unique power that yields the linear "intensity truth-telling" $v_i^* \propto \Pi\theta_i$ and hence utilitarian aggregation.

Annex 2 Quantum ranked-voting

Annex 2.1 Notations

Basic notations		
Notion	Definition	Description
Pure state	$\lvert\psi\rangle = \sum_i c_i \lvert u_i\rangle \in \mathcal{H}$	
Mixed state - density operator	$\rho = \sum_k p_k \lvert\psi_k\rangle \langle\psi_k\rvert \in \mathcal{B}(\mathcal{H})$ $\rho^\dagger = \rho, \;\; \rho \succeq 0, \;\; \mathrm{Tr}\rho = 1$	Here $\mathcal{B}$ is a Banach space
Projector to a subspace $h \subseteq \mathcal{H}$	$\Pi^h = \sum_{\lvert\phi\rangle \in h} \lvert\phi\rangle \langle\phi\rvert, \quad h = \Pi^h \mathcal{H}$	
Support subspace of a density operator	$\mathrm{supp}(\rho) = \mathrm{span}\{\lvert\phi\rangle \in \mathcal{H} \mid \rho\lvert\phi\rangle \neq 0\} \subseteq \mathcal{H}$	
Support of ρ on $h \subseteq \mathcal{H}$	$\mathcal{P}(h) = \mathrm{Tr}(\Pi^h \rho)$	The probability of the measurement outcome of ρ to belong to h
ρ has a support only on h	$\rho = \Pi^h \rho \Pi^h$	

Table 2: Quantum mechanics basic notations

Quantum voting		
Notion	Definition	Description
Preference space	$\mathcal{H}_A := \mathrm{span}(\Gamma), \quad \Gamma = \{\lvert\gamma\rangle : \gamma \in \mathbf{L}(A)\}$	γ's are linear orders on A, and Γ is a preference basis.
Set of density operators on $\mathcal{H}_A$	$\mathcal{D}(\mathcal{H}_A) = \{\rho \in \mathcal{B}(\mathcal{H}_A) \mid \rho^\dagger = \rho, \rho \succeq 0, \mathrm{Tr}(\rho) = 1\}$	Analog of $\mathbf{L}(A)$
Multi-voter joint state	$\sigma_{\mathrm{soc}} \in \mathcal{D}(\mathcal{H}_A^{\otimes n})$	Analog of a profile $P \in \mathbf{L}(A)^n$
Product state (uncorrelated voters)	$\sigma_{\mathrm{soc}} = \rho_1 \otimes ... \otimes \rho_n$	In this case $\rho_i = \sum_k p_k \lvert\psi_k\rangle \langle\psi_k\rvert$
Individual voter's state	$\rho_i = \mathrm{Tr}_{V \setminus i} \sigma_{\mathrm{soc}} \in \mathcal{D}(\mathcal{H}_A)$	
Subspace of partial linear order $l(A)$	$\mathcal{G}^{l(A)} = \mathrm{span}\{\lvert\gamma\rangle \in \mathbf{L}(A) : l(A)\} \subseteq \mathcal{H}_A$	
Pair preference subspace	$\mathcal{G}^{a\Theta b} = \mathrm{span}\{\lvert\gamma\rangle \in \mathbf{L}(A) : a\Theta b)\}, \;\; \Theta = \{\succsim, \precsim\}$	
Preference basis element	$\mathcal{G}^\gamma = \lvert\gamma\rangle$	
Partial order projector	$\Pi^{\mathcal{G}^{l(A)}} = \sum_{\lvert\gamma\rangle \in \mathcal{G}^{l(A)}} \lvert\gamma\rangle \langle\gamma\rvert, \;\; \mathcal{G}^{l(A)} = \Pi^{\mathcal{G}^{l(A)}} \mathcal{H}_A$	
Quantum SWF	$\mathcal{E}(\sigma_{\mathrm{soc}}) = \rho_{\mathrm{soc}}, \quad \mathcal{E} : \mathcal{D}\left(\mathcal{H}_A^{\otimes N}\right) \to \mathcal{D}(\mathcal{H}_A)$	Convex-linear CPTP map
Quantum SWF measurement	$p_{\mathrm{soc}}(\gamma) = \mathrm{Tr}(\lvert\gamma\rangle \langle\gamma\rvert \rho_{\mathrm{soc}})$	Probability of the linear order γ outcome.

Table 3: Quantum ranked-voting notations

Annex 2.2 Quantum version of Arrow's properties

We follow here the definitions given in a paper by Sun et al. (2021). In the following we use the probability of the preference $l(A)$ for voter i/society, in profile P, given by $\mathcal{P}^P_{i/\text{soc}}(l(A)) = \text{Tr}(\Pi^{l(A)}\rho^P_{i/\text{soc}})$.

1. Quantum Transitivity:
   The measurement of ρ_{soc} in the preference basis, $\Gamma = \{|\gamma\rangle : \gamma \in \mathbf{L}(A)\}$, yields some $|\gamma\rangle$, with probability $\mathcal{P}_{\text{soc}}(\gamma)$, which is transitive, by definition.

2. Quantum Unanimity for any $a, b \in A$:
   - (a) Sharp: $\forall i \in V, \quad \mathcal{P}_i(a\Theta b) = 1 \Rightarrow \mathcal{P}_{\text{soc}}(a\Theta b) = 1$
   - (b) Unsharp: $\forall i \in V, \quad \mathcal{P}_i(a\Theta b) > 0 \Rightarrow \mathcal{P}_{\text{soc}}(a\Theta b) > 0$

   A quantum SWF $\mathcal{E}$ satisfies the quantum unanimity condition if it satisfies both sharp and unsharp conditions.

3. Quantum IIA (QIIA):
   Given any two quantum preference profiles – multi-voter joint states, σ_{soc}, and σ'_{soc}, and the resulting society's preferences $\rho_{\text{soc}} = \mathcal{E}(\sigma_{\text{soc}})$, and $\rho'_{\text{soc}} = \mathcal{E}(\sigma'_{\text{soc}})$,
   - (a) Sharp: If $\forall i \in V, \; \mathcal{P}_i(a\Theta b) = \mathcal{P}'_i(a\Theta b)$, then $\mathcal{P}_{\text{soc}}(a\Theta b) = 1$ implies that $\mathcal{P}'_{\text{soc}}(a\Theta b) = 1$
   - (b) Unsharp: If $\forall i \in V, \; \mathcal{P}_i(a\Theta b) = \mathcal{P}'_i(a\Theta b)$, then $\mathcal{P}_{\text{soc}}(a\Theta b) > 0$ implies that $\mathcal{P}'_{\text{soc}}(a\Theta b) > 0$

   A quantum SWF $\mathcal{E}$ satisfies the QIIA condition if it satisfies both sharp and unsharp conditions.

4. Quantum Dictatorship:
   - (a) Sharp: $\exists i, \forall P \in \mathbf{L}(A)^n, \quad \mathcal{P}^P_i(a\Theta b) = 1 \quad \Leftrightarrow \quad \mathcal{P}^P_{\text{soc}}(a\Theta b) = 1$
   - (b) Unsharp: $\exists i, \forall P \in \mathbf{L}(A)^n, \quad \mathcal{P}^P_i(a\Theta b) > 0 \quad \Leftrightarrow \quad \mathcal{P}^P_{\text{soc}}(a\Theta b) > 0$

   A quantum SWF $\mathcal{E}$ satisfies the quantum dictatorship condition if it satisfies both sharp and unsharp conditions.

Annex 2.3 Quantum SWFs

In both cases of quantum SWFs, quantum majority rule, $\mathcal{E}_{\text{QMR}}$, suggested by Bao and Yunger Halpern (2017), and quantum Condorcet voting, $\mathcal{E}_{\text{QCV}}$, proposed by Sun et al. (2021), the authors start with the decoherence of voters' preferences,

$$\rho_i \mapsto \rho'_i := \sum_{\gamma_i} |\gamma_i\rangle\langle\gamma_i|\,\rho_i\,|\gamma_i\rangle\langle\gamma_i| = \sum_{\gamma} p^\gamma_i \chi^\gamma_i \tag{1}$$

where $\chi^\gamma_i = |\gamma_i\rangle\langle\gamma_i|$, $p^\gamma_i = \langle\gamma_i|\,\rho_i\,|\gamma_i\rangle$, and $\sum_\gamma p^\gamma_i = 1$. This gives the product state society's quantum profile,

$$\sigma_{\text{soc}} \mapsto \sigma'_{\text{soc}} = \rho'_1 \otimes \cdots \otimes \rho'_N = \sum_{\gamma_1,\ldots,\gamma_N} (p^{\gamma_1}_1 \ldots p^{\gamma_N}_N)(\chi^{\gamma_1}_1 \otimes \cdots \otimes \chi^{\gamma_N}_N) = \sum_{\mathbf{g}} p(\mathbf{g})\boldsymbol{\chi}(\mathbf{g}) \tag{2}$$

where $\mathbf{g} = (\gamma_1, ..., \gamma_N)$, $p(\mathbf{g}) = p^{\gamma_1}_1 \ldots p^{\gamma_N}_N$, and $\boldsymbol{\chi}(\mathbf{g}) = \chi^{\gamma_1}_1 \otimes \cdots \otimes \chi^{\gamma_N}_N \in \mathcal{H}^{\otimes n}_A$. Hence, in both cases, the quantum SWF acts on the product of preference basis elements, $\boldsymbol{\chi}(\mathbf{g})$,

$$\mathcal{E}_{\text{QMR/QCV}}(\sigma'_{\textbf{soc}}) = \sum_{\mathbf{g}} p(\mathbf{g})\mathcal{E}_{\text{QMR/QCV}}\left(\boldsymbol{\chi}^{\mathbf{g}}\right) = \sum_{\mathbf{g}} p(\mathbf{g})\chi^{\text{QMR/QCV}}_{\text{soc}}(\mathbf{g}) = \rho^{\text{QMR/QCV}}_{\text{soc}} \in \mathcal{H}_A \tag{3}$$

Annex 2.3.1 Quantum Majority Rule $\mathcal{E}_{\mathbf{QMR}}$

1. Digraph
   (a) Pairwise tallies $n_{ab}(\mathbf{g}) = |\{i \in V : a \succsim_i^{\mathbf{g}} b \wedge \neg(b \succsim_i^{\mathbf{g}} a)\}|$
   (b) Majority relation $\mu_{ab}(\mathbf{g}) = n_{ab}(\mathbf{g}) - n_{ba}(\mathbf{g})$ corresponds the majority preference in a certain $\mathbf{g}$,
   $$\mu_{ab}(\mathbf{g}) > 0 \quad \Leftrightarrow \quad a \succ_M^{\mathbf{g}} b, \qquad \mu_{ab}(\mathbf{g}) = 0 \quad \Leftrightarrow \quad a \sim_M^{\mathbf{g}} b$$
   (c) Edges matrix
   $$E_{ab}(\mathbf{g}) = \begin{cases} 1 & a \rightarrow b \\ 0 & a \leftrightarrow b \\ -1 & a \leftarrow b \end{cases} \tag{4}$$
   where $E_{ab} = \frac{\mu_{ab}}{|\mu_{ab}|}$. We have $E_{ab} = 1$ if more voters prefer a to b, which is represented by the edge from a to b, $E_{ab} = -1$ if opposite, and $E_{ab} = 0$, in the case of tie.
   (d) Digraph $G = (A, \mathbf{E})$ with alternatives as vertices and edges determined above.
2. *Strongly Connected Components* (SCC)
   The set of longest connected vertices, called SCCs, $\mathcal{C}_i$'s, is denoted as $\mathcal{C}(\mathbf{g}) = \{\mathcal{C}_1, ..., \mathcal{C}_k\}$, where every SCC represents a cycle.
3. Directed acyclic graph, DAG $= (V^*, \mathbf{E}^*)$, is defined by SCCs as vertices $V^* = \mathcal{C}$, and an adjacency matrix $\mathbf{E}^*$ of size $k \times k$
   $$E^*_{ij} = \begin{cases} 1 & \exists u \in \mathcal{C}_i, v \in \mathcal{C}_j : E_{uv} = 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$
4. Popularity order
   The DAG structure allows to construct the popularity order of SCCs, $\mathcal{C}^*(\mathbf{g}) = \{\mathcal{C}^*_1, ..., \mathcal{C}^*_k\}$, where the less popular SCC is, the later it appears in the list.
5. Maximally mixed societal state
   (a) Set of strict linear orders $\mathcal{L}(\mathbf{g}) = \{\gamma \in \mathbf{L}(A) : a \in \mathcal{C}^*_j(\mathbf{g}), b \in \mathcal{C}^*_i(\mathbf{g}), j \succ i \Rightarrow a \succ_\gamma b\}$
   (b) $\chi^{\mathrm{QMR}}_{(1)}(\mathbf{g}) = \frac{1}{|\mathcal{L}(\mathbf{g})|} \sum_{\gamma \in \mathcal{L}(\mathbf{g})} |\gamma\rangle\langle\gamma|$

Annex 2.3.2 Quantum Condorcet Voting $\mathcal{E}_{\mathbf{QCV}}$

1. Condorcet score for a certain candidate, $S_x^{\mathbf{g}} = |\{y \in A : \sum_i \mathrm{Tr}(\Pi_i^{x\succ y}\chi_i^{\gamma_i}) \geq \sum_i \mathrm{Tr}(\Pi^{y\succ x}\chi_i^{\gamma_i})\}|$
   $S : D(\mathcal{H}_A^{\otimes n}) \rightarrow \mathbb{R}$
2. Condorcet scores vector $\mathbf{S}^{\mathbf{g}} = (S^{\mathbf{g}}_{x,\max}, ..., S^{\mathbf{g}}_{x,\min})$
3. Complete weak order according to the Condorcet scores, $R_S^{\mathbf{g}}(\boldsymbol{\chi}^{\mathbf{g}}) = (x_{\max}, ..., x_{\min})$, (may contain ties)
   $CS : D(\mathcal{H}_A^{\otimes n}) \rightarrow R(A)$
4. Linear order extension of a weak order
   (a) Extension of a weak relation: $x \sim y \Rightarrow \{(x, y), (y, x)\}$
   (b) Number of ties $T^{\mathbf{g}}_{CS} = |\{(x, y) \in R^{\mathbf{g}}_S \wedge (y, x) \in R^{\mathbf{g}}_S\}|$
   (c) Linear extension of weak order $LE(R^{\mathbf{g}}_S) = L^{\mathbf{g}}_E = \{s \subseteq R^{\mathbf{g}}_S | s$ is complete, transitive and irreflexive$\}$

   For every tie in a weak order $(x, y) \in R^{\mathbf{g}}_S \wedge (y, x) \in R^{\mathbf{g}}_S$, the extension linear order $L^{\mathbf{g}}_E$ contains a pair of both $x \succ y$ and $y \succ x$, where $|L_E| = |R^P_C|T_C!$.
   Example: $R = (a = b > c) \Rightarrow L_E = \{(a > b > c), (b > a > c)\}$

   $LE : R(A) \rightarrow L(A)$
5. Quantum extension state $\chi^{\mathrm{QCV}}_{(1)}(\mathbf{g}) = \frac{1}{|L^{\mathbf{g}}_E|} \sum_{l \in L^{\mathbf{g}}_E} |l\rangle\langle l|$.
   $QE : L(A) \rightarrow D(\mathcal{H}_{\mathcal{A}})$

Annex 2.3.3 Give the minority a shot and enforce unanimity

1. Give the minority a shot (GMS)

$$\chi_{(2)}(\mathbf{g}) = \mathrm{GMS}\left(\chi_{(1)}(\mathbf{g})\right) = (1 - |R_{\mathrm{supp}}(\mathbf{g})|\delta)\chi_{(1)}(\mathbf{g}) + \delta \sum_{(a,b)\in R_{\mathrm{supp}}(\mathbf{g})} \frac{\Pi^{a\succ b}}{\mathrm{Tr}(\Pi^{a\succ b})} \tag{6}$$

   where $0 < \delta \ll 1$, and $R_{\mathrm{supp}}(\mathbf{g}) = \left\{(a,b) : \mathrm{Tr}\Big(\Pi^{a\succ b}\boldsymbol{\chi}(\mathbf{g})\Big) > 0\right\}$ is the set of all the preferences existing in $\boldsymbol{\chi}(\mathbf{g})$.
   If $\forall(a,b) \in R_{\mathrm{supp}}(\mathbf{g}),\ \ \mathrm{Tr}(\Pi^{a\succ b}\chi_{(1)}(\mathbf{g})) = 0$, then $\mathrm{Tr}\left(\Pi^{a\succ b}\chi_{(2)}(\mathbf{g})\right) \geq \delta > 0$.

   This step spoils the unanimity property.

2. Enforcing unanimity (EU)

   (a) Unanimity set $U(\mathbf{g}) = \{(a,b) : \forall i, \mathrm{Tr}(\Pi^{\succ b}\rho'_i(\mathbf{g})) = 1\}$
   (b) Unanimity projector $\Pi^{\mathrm{U}(\mathbf{g})} = \prod_{(a,b)\in U(\mathbf{g})} \Pi^{a\succ b}$
   (c) Society's preference satisfying unanimity

$$\chi_{(3)}(\mathbf{g}) = \mathrm{EU}(\chi_{(2)}(\mathbf{g})) = \frac{\Pi^{U(\mathbf{g})}\chi_{(2)}(\mathbf{g})\Pi^{U(\mathbf{g})}}{\mathrm{Tr}\left(\Pi^{U(\mathbf{g})}\chi_{(2)}(\mathbf{g})\right)} \tag{7}$$

   If one is not interested in GMS, then the EU step has no influence, $\mathrm{EU}(\chi_{(1)}(\mathbf{g})) = \chi_{(1)}(\mathbf{g})$.

3. Final social preference state

$$\rho_{\mathrm{soc}}^{\mathrm{QMR/QCV}} = \sum_{\mathbf{g}} p(\mathbf{g})\chi_{(3)}^{\mathrm{QMR/QCV}}(\mathbf{g}) \tag{8}$$

Annex 2.3.4 Election result

The outcome of the quantum SWF is obtained by the measurement of ρ_{soc} in the preference basis $\Gamma = \{|\gamma\rangle : \gamma \in \mathbf{L}(\mathbf{O})\}$. With probability $p_{\mathrm{soc}}(\gamma) = \mathrm{Tr}(|\gamma\rangle\langle\gamma|\,\rho_{\mathrm{soc}})$, the outcome of the SWF is the strict linear order γ.

References

Aharonov, Y., Cohen, E., and Landsberger, T. (2017). The two-time interpretation and macroscopic time-reversibility. *Entropy*, 19(3):111.

Aristotle (1996). *Politics.* Cambridge University Press, Cambridge. Trans. B. Jowett.

Arrow, K. J. (1951). *Social choice and individual values.* Wiley, New York.

Balinski, M. and Laraki, R. (2007). A theory of measuring, electing, and ranking. *Proceedings of the National Academy of Sciences*, 104(21):8720–8725.

Bao, N. and Yunger Halpern, N. (2017). Quantum voting and violation of arrow's impossibility theorem. *Physical Review A*, 95(6):062306.

Bassetti, M. E., Dias, G., Chen, D. L., Mortoni, A., and Das, R. (2023). Civicbase: An open-source platform for deploying quadratic voting for survey research. *AI Magazine*, 44(3):263–273.

Beetham, D. (1991). *The legitimation of power.* Macmillan, London.

Bejan, A. (1997). Constructal-theory network of conducting paths for cooling a heat generating volume. *International Journal of Heat and Mass Transfer*, 40(4):799–816.

Bejan, A. (2000). *Shape and Structure, from Engineering to Nature.* Cambridge University Press, Cambridge.

Borda, J.-C. d. (1781). Mémoire sur les élections au scrutin. *Histoire de l'Académie Royale des Sciences*, pages 657–665.

Bowen, J. R. (1986). On the political construction of tradition: Gotong royong in indonesia. *The Journal of Asian Studies*, 45(3):545–561.

Brams, S. J. and Fishburn, P. C. (2002). Voting procedures. In Arrow, K. J., Sen, A. K., and Suzumura, K., editors, *Handbook of social choice and welfare*, volume 1, pages 173–236. Elsevier.

Brams, S. J. and Fishburn, P. C. (2007). *Approval voting.* Springer, New York, 2 edition.

Buterin, V., Hitzig, Z., and Weyl, E. G. (2018). Liberal radicalism: A flexible design for philanthropic matching funds. `https://arxiv.org/abs/1809.06421`. arXiv preprint arXiv:1809.06421.

Byock, J. L. (2001). *Viking age Iceland.* Penguin, London.

Casella, A. and Sanchez, L. (2019). Storable votes and quadratic voting: An experiment on four california propositions. Technical Report 25510, National Bureau of Economic Research.

Celano, A. J. (2024). Interpreting aristotle's concept of the common good. In Gallagher, R. S. and Sison, A. A. M., editors, *The Common Good: An Introduction to Personalism*, pages 49–65. Springer, Cham.

Chafuen, A. A. (2003). *Faith and Liberty: The Economic Thought of the Late Scholastics.* Lexington Books, Lanham, MD.

Cheng, T.-C., Li, T.-W., Chou, Y.-H., Karahalios, K., and Sundaram, H. (2021). I can show what i really like: Eliciting preferences via quadratic voting. *Proceedings of the ACM on Human-Computer Interaction*, 5(CSCW1):182.

Cheng, T.-C., Zhang, Y., Chou, Y., Koshy, V., Li, T., Karahalios, K., and Sundaram, H. (2025). Organize, then vote: Exploring cognitive load in quadratic survey interfaces. In *Proceedings of the 2025 CHI Conference on Human Factors in Computing Systems*, pages 1–35.

Cohen, E. (n.d.). A final boundary condition: Several implications for the universe. Public lecture, Perimeter Institute, Ontario, Canada.

Colomer, J. M. (2012). Ramon llull: From *ars combinatoria* to social choice theory. *Social Choice and Welfare*, 38:1–15.

Colomer, J. M. (2013). Ramon llull: From ars electionis' to social choice theory. *Social Choice and Welfare*, 40:317–328.

Condorcet, M. J. A. N. d. C. (2014). *Essay on the application of analysis to the probability of majority decisions.* Edward Elgar, Cheltenham, UK. Original work published 1785, translated by Iain McLean & Christine Blundell.

Council of the European Union (2024). Qualified majority. `https://www.consilium.europa.eu/en/council-eu/voting-system/qualified-majority/`.

Dahl, R. A. (1989). *Democracy and its critics.* Yale University Press, New Haven, CT.

Dryzek, J. S. (2000a). *Deliberative democracy and beyond: Liberals, critics, contestations.* Oxford University Press, Oxford.

Dryzek, J. S. (2000b). *Deliberative democracy and beyond: Liberals, critics, contestations.* Oxford University Press, Oxford.

Edelman (2024). 2024 edelman trust barometer: Global report. `https://tinyurl.com/3fdereus`.

Equal Vote Coalition (2019). Star voting: An overview. `https://www.starvoting.org`.

European City Squared Consortium (2025). From quantum democracy to spacetime geometry. Unpublished Working paper https://tinyurl.com/mr2asj9k. Accessed 2025.

European City Squared Project (2025). European city squared: Computational social science simulation for democracy. Horizon Europe Programme, Grant Agreement No. 101178170 https://cordis.europa.eu/project/id/101178170.

European Commission (2023). Defence of democracy package. Technical Report COM(2023) 630 final, European Commission, Brussels.

Fabinger, M., Freedman, M. H., and Weyl, E. G. (2022). Prospecting a possible quadratic wormhole between quantum mechanics and plurality. `https://arxiv.org/abs/2209.08144`.

Farrell, D. M. (2011). *Electoral systems: A comparative introduction.* Palgrave Macmillan, Basingstoke, 2 edition.

Fishkin, J. S. (2009). *When the people speak: Deliberative democracy and public consultation.* Oxford University Press, Oxford.

Fortescue, J. (1997). *On the Laws and Governance of England.* Cambridge University Press, Cambridge.

Freedom House (2023). The global expansion of authoritarian rule. Technical report, Freedom House, Washington, DC.

Friston, K. (2010). The free-energy principle: A unified brain theory? *Nature Reviews Neuroscience*, 11(2):127–138.

Gehrlein, W. V. (2006). *Condorcet's paradox.* Springer, Berlin.

Gibbard, A. (1973). Manipulation of voting schemes: A general result. *Econometrica*, 41(4):587–601.

Gilbert, J.-É. and Marijuan, P. (2001). Ramon llull: The first theorist of voting. *History of Political Thought*, 22(1):25–50.

Gitcoin (2023). Driving innovation with quadratic voting: Fantom's journey with grants stack. Gitcoin Blog, https://www.gitcoin.co/blog/innovation-with-qv-fantom-a-grants-stack-case-study.

Goeree, J. K. and Zhang, J. (2017). One man, one vote: Efficiency and equity of quadratic voting. *Public Choice*, 172(1-2):129–153.

Grice-Hutchinson, M. (1952). *The School of Salamanca: Readings in Spanish Monetary Theory, 1544–1605.* Clarendon Press, Oxford.

Gudynas, E. (2011). Buen vivir: Today's tomorrow. *Development*, 54(4):441–447.

Hägele, G. and Pukelsheim, F. (2001). Llull's writings on electoral systems. *Studia Lulliana*, 41(97):3–38.

Hirschman, A. O. (1977). *The Passions and the Interests: Political Arguments for Capitalism before Its Triumph.* Princeton University Press, Princeton.

Hong, L. and Page, S. E. (2004). Groups of diverse problem solvers can outperform groups of high-ability problem solvers. *Proceedings of the National Academy of Sciences*, 101(46):16385–16389.

Hortala-Vallve, R. (2012). Qualitative voting. *Journal of Theoretical Politics*, 24:526–555.

Kaplow, L. and Kominers, S. D. (2017). Who will vote quadratically? voter turnout and votes cast under quadratic voting. *Public Choice*, 172(1):125–149.

Kornhauser, L. A. and Sager, L. G. (1986). Unpacking the court. *Yale Law Journal*, 96(1):82–117.

Koselleck, R. (2004). *Futures Past: On the Semantics of Historical Time.* Columbia University Press, New York.

Kovalchuk, L., Rodinko, M., Oliynykov, R., Nastenko, A., Kaidalov, D., and Nelson, K. (2025). Enhancing decentralization in blockchain decision-making through quadratic voting and its generalization. `https://doi.org/10.48550/arXiv.2504.12859`. arXiv:2504.12859.

Kowalchuk, L., Rodinko, M., Oliynykov, R., Nastenko, A., Kaidalov, D., and Nelson, K. (2025). Universally composable on-chain quadratic voting for liquid democracy. `https://eprint.iacr.org/2025/803`.

Krakauer, D. C. (2024). *The Complex World: An Introduction to the Foundations of Complexity Science.* The SFI Press Scholars Series. SFI Press, Santa Fe, NM.

Lalley, S. P. and Weyl, E. G. (2017). Quadratic voting: How mechanism design can radicalize democracy. *AEA Papers and Proceedings*, 107(5):1–5.

Lalley, S. P. and Weyl, E. G. (2018). Quadratic voting: How mechanism design can radicalize democracy. *AEA Papers and Proceedings*, 108:33–37.

Landemore, H. (2013). *Democratic reason: Politics, collective intelligence, and the rule of the many.* Princeton University Press, Princeton, NJ.

Levin, M. (2014). Molecular bioelectricity: How endogenous voltage potentials control cell behavior and instruct pattern regulation in vivo. *Molecular Biology of the Cell*, 25(24):3835–3850.

Levin, M. (2021). Bioelectric signaling: Reprogrammable circuits underlying embryogenesis, regeneration, and cancer. *Cell*, 184(8):1971–1989.

Lijphart, A. (2012). *Patterns of democracy: Government forms and performance in thirty-six countries.* Yale University Press, New Haven, CT, 2 edition.

Lindholt, M. F., Jørgensen, F., Bor, A., and Petersen, M. B. (2023). Why trust? a mixed-method investigation of the origins and meaning of trust during the covid-19 lockdown in denmark. *Social Science & Medicine*, 317:115568.

List, C. (2013). Social choice theory. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy (Fall 2013 Edition).* Metaphysics Research Lab, Stanford University.

Lorenz, J. (2007). Continuous opinion dynamics under bounded confidence: A survey. *International Journal of Modern Physics C*, 18(12):1819–1838.

Lorenz, J., Rauhut, H., Schweitzer, F., and Helbing, D. (2011a). How social influence can undermine the wisdom of crowd effect. *Proceedings of the National Academy of Sciences*, 108(22):9020–9025.

Lorenz, J., Rauhut, H., Schweitzer, F., and Helbing, D. (2011b). How social influence can undermine the wisdom of crowd effect. *Proceedings of the National Academy of Sciences*, 108(22):9020–9025.

Lorenz, J., Rauhut, H., Schweitzer, F., and Helbing, D. (2011c). How social influence can undermine the wisdom of crowd effect. *Proceedings of the National Academy of Sciences*, 108(22):9020–9025.

Machiavelli, N. (1998). *The Prince.* University of Chicago Press, Chicago, 2 edition. Original work published 1532.

Mariana, J. d. (1599). *De rege et regis institutione.* [Publisher not identified]. Early modern edition.

Markowski, R. and Kotnarowski, M. (2025). Alternatives to liberal democracies and their consolidation in central and eastern europe. *Politics and Governance*, 13.

Marx, K. (1990). *Capital: A Critique of Political Economy, Volume I.* Penguin Classics, London. Original work published 1867; translated by Ben Fowkes.

McLean, I. and Urken, A. B. (1995). *Classics of Social Choice.* University of Michigan Press, Ann Arbor.

Metz, T. (2011). Ubuntu as a moral theory and human rights in south africa. *African Human Rights Law Journal*, 11(2):532–559.

Mill, J. S. (1998). *Considerations on representative government.* Oxford University Press, Oxford. Original work published 1861.

Molina, L. d. (1593). *De iustitia et iure.* [Publisher not identified]. Early modern edition.

Molina, L. d. (1614). *De Iustitia et Iure.* [Publisher not identified]. Early modern edition.

Morrison, D. (2013). The common good. In Deslauriers, M. and Destrée, P., editors, *The Cambridge Companion to Aristotle's* Politics, pages 176–198. Cambridge University Press, Cambridge.

Mozumder, N. A. (2022). Can ethical political leadership restore public trust in political leaders? *Public Organization Review*, 22:821–835.

Murtin, F. et al. (2018). Trust and its determinants: Evidence from the trustlab experiment. OECD Statistics Working Papers 2018/02, OECD.

Nagel, T. (1999). *The Law of Peoples.* Harvard University Press, Cambridge, MA.

Nelson, K. P., Attieh, J., and Oliveira, I. (2024). Simulating the quality of community decisions. Technical Report SNET DF4B-RFP2, Photrek, LLC.

Norris, P. (2014). *Why electoral integrity matters.* Cambridge University Press, Cambridge.

Parkinson, J. (2006). *Deliberating in the real world: Problems of legitimacy in deliberative democracy.* Oxford University Press, Oxford.

Parr, T. and Friston, K. J. (2019). Generalised free energy and active inference: Can the future cause the past? *Trends in Cognitive Sciences*, 23(10):727–736.

Penrose, L. S. (1946). The elementary statistics of majority voting. *Journal of the Royal Statistical Society*, 109(1):53–57.

Peter, F. (2023). Political legitimacy. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy (Winter 2023 Edition).* Metaphysics Research Lab, Stanford University.

PMC (PubMed Central) (2022). A systematic literature review and meta-analysis on scalable blockchain-based electronic voting systems. PMC. Updated 2025 case study.

Posner, E. A. and Weyl, E. G. (2015). Voting squared: Quadratic voting in democratic politics. *Vanderbilt Law Review*, 68(2):441–483.

Posner, E. A. and Weyl, E. G. (2018). *Radical Markets: Uprooting Capitalism and Democracy for a Just Society.* Princeton University Press, Princeton.

Public Choice (2017). Quadratic voting in the wild: Real people, real votes. Published July 2017.

Putnam, R. D. (2000). *Bowling alone: The collapse and revival of American community.* Simon & Schuster, New York.

Razo, L. (2014). Voluntary taxation and the future of democracy. In Bañón i Martínez, R. and Tamboleo García, R., editors, *La modernización de la gestión y asuntos públicos*, pages 110–131. Editorial GOGEP Complutense, Madrid.

Reynolds, A., Reilly, B., and Ellis, A. (2005). *Electoral system design: The new International IDEA handbook.* International

Institute for Democracy and Electoral Assistance, Stockholm.

Rogers, A. (2019). Colorado tried a new way to vote: Make people pay—quadratically. Wired, April 16, 2019.

Rovelli, C. (1996). Relational quantum mechanics. *International Journal of Theoretical Physics*, 35(8):1637–1678.

Rovelli, C. (2022). *Helgoland.* Riverhead Books, New York.

Saari, D. G. (1995). *Basic geometry of voting.* Springer, Berlin.

Satterthwaite, M. A. (1975). Strategy-proofness and arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2):187–217.

Scharpf, F. W. (1999). *Governing in Europe: Effective and democratic?* Oxford University Press, Oxford.

Schumpeter, J. A. (1942). *Capitalism, socialism, and democracy.* Harper & Brothers, New York.

Sen, A. (1999). The possibility of social choice. *American Economic Review*, 89(3):349–378.

Smith, A. (1937). *An Inquiry into the Nature and Causes of the Wealth of Nations.* Modern Library, New York. Original work published 1776; edited by Edwin Cannan.

Smith, G. (2009). *Democratic innovations: Designing institutions for citizen participation.* Cambridge University Press, Cambridge.

Smith, W. D. (2000). Range voting. *Social Choice and Welfare*, 17:381–388.

Sun, X., He, F., Sopek, M., and Guo, M. (2021). Schrödinger's ballot: Quantum information and the violation of arrow's impossibility theorem. *Entropy*, 23(8):1083.

Sun, X., Su, X., and Bian, X. (2025). Quantum vote based on quantum logical operators. *Quantum Information Processing*, 24:234.

Svendsen, G. T. and Svendsen, G. L. H. (2016). Trust, social capital and the scandinavian welfare state. In Uhlaner, A., Lutz, B., and Tidemann, L., editors, *Handbook on the Economics of Reciprocity and Social Enterprise.* Edward Elgar, Cheltenham.

Taylor, A. D. (2005). *Social choice and the mathematics of manipulation.* Cambridge University Press, Cambridge.

The Guardian (2024). The denmark secret: How it became the world's most trusting country – and why that matters. `https://tinyurl.com/23zvduw3`. News article.

Tideman, T. N. (1995). The single transferable vote. *Journal of Economic Perspectives*, 9(1):27–38.

Tideman, T. N. and Plassmann, F. (2011). Modeling the outcomes of vote-casting in actual elections. `https://ssrn.com/abstract=1627787`.

Tierney, B. (1997). *The Idea of Natural Rights: Studies on Natural Rights, Natural Law, and Church Law, 1150–1625.* Eerdmans, Grand Rapids, MI.

Tierney, B. (1998). *Foundations of the Conciliar Theory: The Contribution of the Medieval Canonists from Gratian to the Great Schism.* Brill, Leiden. Original work published 1955.

Tørnqvist, M. (2023). Nordic exceptionalism under pressure: Polarization and trust in the 2020s. *Nordic Political Studies.*

Tyler, T. R. (2006). *Why people obey the law.* Princeton University Press, Princeton, NJ, 2 edition.

Ullah, A. A. (2025). Trends in political science research: Democratic dissatisfaction. *International Political Science Abstracts*, 75(3):391–407.

Vitoria, F. d. (1991). *Political Writings.* Cambridge University Press, Cambridge. Edited by Anthony Pagden and Jeremy Lawrance.

Wellings, T., Banaie Heravan, F., Sharma, A., Gelauff, L., Fricker, R. H., and Pournaras, E. (2024). Fair and inclusive participatory budgeting: Voter experience with cumulative and quadratic voting interfaces. In Bramwell-Dicks, A., Evans, A., Winckler, M., Petrie, H., and Abdelnour-Nocera, J., editors, *Design for Equality and Justice. INTERACT 2023*, volume 14536 of *Lecture Notes in Computer Science*. Springer, Cham.

Weyl, E. G. (2012). Quadratic vote buying. Working paper.

Weyl, E. G., Frederickson, M. E., Yu, D. W., and Pierce, N. E. (2010). Economic contract theory tests models of mutualism. *Proceedings of the National Academy of Sciences of the United States of America*, 107(36):15712–15716.